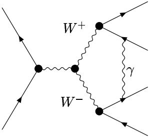
Four-fermion production near the W pair production threshold with unstable particle effective field theory

M. Beneke (RWTH Aachen)

Outline

- Introduction
- Unstable particle EFT and the Born cross section
- Radiative corrections
- Result, δM_W
- Dominant NNLO corrections and cuts



MB, Chapovsky, Signer, Zanderighi, PRL93:011602,2004; NPB686:205-247,2004 (EFT formalism)
MB, Kauer, Signer, Zanderighi, Nucl.Phys.Proc.Suppl.152:162-167,2006 (WW)
MB, Falgari, Schwinn, Signer, Zanderighi, NPB792:89, 2008 (0707.0773 [hep-ph]) (NLO WW threshold)
Actis, MB, Falgari, Schwinn, 0807.0102 [hep-ph] (WW threshold beyond NLO)

Motivation

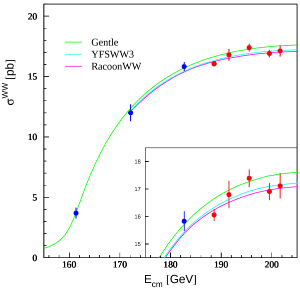
- "Fundamental question in QFT" – Perturbation expansions do not work for the production of resonances ("unstable particles") even for weak coupling, because

$$M^2/(s - M^2) \sim M^2/(M\Gamma) \sim 1/g^2$$

Systematic expansion?

- The electroweak gauge bosons W, Z, the top quark and, perhaps, the Higgs boson (if $m_H \geq 2m_W$) decay rapidly ($\tau < 10^{-25}s$) such that

$$\frac{\text{width}}{\text{mass}} \equiv \frac{\Gamma}{M} \sim \mathcal{O}(\alpha_{\text{EW}}) \ll 1 \qquad \text{but non-negligible.}$$



- "Electroweak precision tests" – Measurements of M_W and m_t determine M_H or M_{new} through virtual effects. Accurate W mass from

$$pp \longrightarrow WX \longrightarrow \ell\nu X$$
$$e^-e^+ \longrightarrow W^+W^-X \longrightarrow \mu^-\bar{\nu}_\mu u\bar{d}\,X$$

What's the problem?

- "Kinematical" breakdown of perturbation theory. Propagators become singular near resonance

$$\frac{g^2}{p^2 - M^2 + i\epsilon} \sim 1 \quad \text{when} \quad p^2 - M^2 \sim M\Gamma \sim (gM)^2$$

  Process involves two very different scales: short-distance production ($1/\sqrt{s}$, $1/M$) and the lifetime $1/\Gamma \gg 1/M$ (unless the contour can be deformed away from the singularity).

- "Dyson resummation" of self-energy insertions

$$\frac{1}{s - M^2} \rightarrow \frac{1}{s - M^2 - \Pi(s)}$$

  regularizes the singularity, since $\Pi(M^2) \approx \delta M^2 - iM\Gamma$, but: gauge-dependence of $\Pi(s)$ and the propagator of a gauge boson resonance.
  Need a systematic approximation in g^2 and Γ/M to the scattering amplitude/cross section.

- Note: unstable particles have no asymptotic states and their lines are never cut in Cutkosky's rules (*Veltman, 1963*). Theory is unitary in the Hilbert space of asymptotic states. "On-shell" production of unstable particles corresponds to the leading-order approximation

$$\frac{M\Gamma}{(p^2 - M^2) + M^2\Gamma^2} \overset{\Gamma \to 0}{\longrightarrow} \pi\delta(p^2 - M^2)$$

Methods/approaches

Mainly to deal with gauge invariance. Often more pragamtic than systematic.

- "Fermion-loop scheme" (*Argyres et al., 1995*)
- "Pinch technique" (*Papavassilou et al., 1994*)
- "(Double) Pole approximation" (*Stuart, 1991; Aeppli, van Oldenborgh, Wyler, 1994*)

  Expansion of scattering amplitude around the complex pole of the resonance(s).

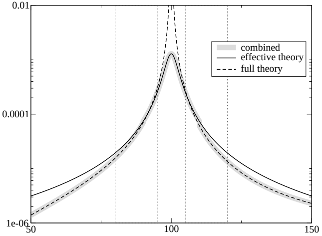
  - Exploits $\Gamma \ll M$.
  - Diagrammatic, never beyond NLO.
  - Breaks down for pair production near threshold (?)
- "Complex mass scheme" (*Denner, Dittmaier, Roth, Wackeroth, 1999*)
  - Standard perturbative calculation with complex mass counterterms, so $p^2 - M^2$ is never zero.
  - With M_Z, M_W and G_F as inputs for the renormalized electroweak parameters $\longrightarrow \sin\theta_W$ and coupling constants become complex (essential for Ward identities to hold).

Complete NLO calculation of $e^- e^+ \to 4f$ has been performed (*Denner, Dittmaier, Roth, Wieders, 2005*) in the complex mass scheme.
Rather challenging – first 1-loop calculation of a $2 \to 4$ process.

Matching kinematic regions

- Consider line-shape $A + B \longrightarrow \text{resonance} \longrightarrow X$

$$\delta \equiv \frac{s - M^2}{m^2}$$



- Off resonance, $\delta \sim 1$, conventional perturbation theory applies

$$\sigma = g^4 f_1(\delta) + g^6 f_2(\delta) + \ldots$$

- Near resonance, $\delta \ll 1$, expand in δ and reorganize

$$\sigma \sim \sum_n \left(\frac{g^2}{\delta}\right)^n \times \{1\,(\text{LO}); g^2, \delta\,(\text{NLO}), \ldots\} = h_1(g^2/\delta) + g^2 h_2(g^2/\delta) + \ldots$$

- The two approximations can be matched in an intermediate region, where δ and g^2/δ are small.
  In the following we concentrate on the resonance region (threshold for pair production).

Inclusive $e^- e^+ \to 4f$

- Consider

$$e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d}\, X$$

near threshold. Dominated by nearly on-shell $W^- W^+$. Large sensitivity to M_W.
ILC with GIGAZ option: $\delta M_W \approx 6\,\text{MeV}$ experimentally (Wilson, 2001).
Rule of thumb: $\delta\sigma \approx 1\% \Leftrightarrow \delta M_W \approx 15\,\text{MeV}$.

- Calculate totally inclusive final state, except for flavour quantum numbers.
Extract cross section from the forward-scattering amplitude

$$\hat{\sigma} = \frac{1}{s}\,\text{Im}\,\mathcal{A}(e^- e^+ \to e^- e^+)_{|\mu^- \bar{\nu}_\mu u \bar{d}}$$

- Perform a “QCD-style” calculation of the short-distance cross section with massless electrons in the $\overline{\text{MS}}$ scheme, then

$$\sigma(s) = \int_0^1 dx_1 dx_2\, f_{e/e}(x_1)\, f_{e/e}(x_2)\, \hat{\sigma}(x_1 x_2 s).$$

$\overline{\text{MS}}$ electron distribution function depends on m_e, but not on $\sqrt{s}$, M, Γ.

Scales, parameters, power counting – WW and $t\bar{t}$

- WW pair production near threshold is dominated by electroweak interactions (in leading orders), top pair production by the strong interaction.
-

	WW	$t\bar{t}$
α_{ew}	δ (def.)	δ^2
α_{em}	δ	δ^2
α_s	$\sqrt{\delta}$	δ (def.)
Γ/M	δ	δ^2
$v^2 \equiv (\sqrt{s} - [2M + i\Gamma])/M$	δ	δ^2
g^2/v (Coulomb)	$\sqrt{\delta}$	1

- Both require non-relativistic + unstable particle EFT, but for top the former is more essential, while for W unstable particle effects are more important, and the Coulomb interaction does not have to be summed.
  Expansion runs in $\sqrt{\delta}$: LO, $\text{N}^{1/2}$LO, NLO, ...

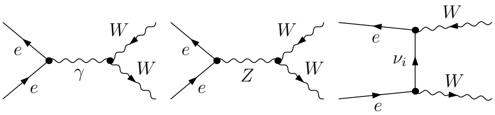
Unstable particle EFT (I)

For simplicity, consider SM with $\alpha_s = 0$.
Integrate out short-distance fluctuations, such that only virtualities $k^2 \ll M_W^2$ are left.
What are the fields and interactions in the EFT?

- Fields
  - No top, Z, Higgs.
  - Two non-relativistic spin-1 fields $\Omega^i_\mp$.
  - Photon and light fermion fields (soft and collinear).
- Interactions
  - The nearly on-shell W bosons can interact with soft photons ($k \sim \Gamma_W$) or potential photons ($k^0 \sim \Gamma$, $\vec{k} \sim \sqrt{M_W \Gamma_W}$) (Coulomb interaction)
  - Soft and collinear ($k^0 \sim M_W$, $k^2 \ll M_W^2$) interactions with the high-energy, initial state electron (positron).
  - The production of the W bosons is short-distance and must be incorporated into the EFT by local operators (more precisely, local modulo collinear Wilson lines).

Unstable particle EFT (II)

Matching of the leading production operator



$$\mathcal{O}_p^{(0)} = \frac{\pi\alpha_{ew}}{M_W^2}\left(\bar{e}_{c2,L}\gamma^{[i}n^{j]}e_{c1,L}\right)\left(\Omega_-^{\dagger i}\Omega_+^{\dagger j}\right)$$

At LO in the expansion around threshold, only the t-channel diagram contributes.

General formula for the forward-scattering amplitude including non-resonant production

$$i\,\mathcal{A} = \sum_{k,l}\int d^4x\,\langle e^+e^-|T(i\mathcal{O}_p^{(k)}(0)i\mathcal{O}_p^{(l)}(x))|e^+e^-\rangle + \sum_k \langle e^+e^-|i\mathcal{O}_{4e}^{(k)}(0)|e^+e^-\rangle.$$

The local four-electron operator includes off-shell WW or single W intermediate states.

Unstable particle EFT (III)

$$\mathcal{L}_{\rm eff} = \sum_{\mp} \left[\Omega_{\mp}^{\dagger i} \left(iD_s^0 + \frac{\vec{\partial}^2}{2M_W} - \frac{\Delta}{2} \right) \Omega_{\mp}^i + \Omega_{\mp}^{\dagger i} \frac{(\vec{\partial}^2 - M_W \Delta)^2}{8M_W^3} \Omega_{\mp}^i \right] + \int d^3 r \left[\Omega_-^{\dagger i} \Omega_-^i (x + \vec{r}) \right] \left(-\frac{\alpha_{\rm QED}}{r} \right) \left[\Omega_+^{\dagger j} \Omega_+^j \right] (x) + \ldots$$

- Δ is a short-distance coefficient determined by matching the W two-point function. Let $\bar{s} \equiv M_W^2 - iM_W \Gamma_W$ be the complex pole position, $\bar{s} - \hat{M}_W^2 - \Pi_T^W(\bar{s}) = 0$. Then

$$\Delta \equiv \frac{\bar{s} - \hat{M}_W^2}{\hat{M}_W} \overset{\text{pole scheme}}{=} -i\Gamma_W$$

- For the massless fields obtain terms familiar from the soft-collinear effective theory (SCET). Not much of this is needed explicitly at NLO.
- Propagator

$$\frac{i\,\delta^{ij}}{\left(k^0 - \frac{\vec{k}^2}{2M_W} - \frac{\Delta^{(1)}}{2} \right)}$$

This accomplishes the reorganisation of PT, since $\Delta^{(1)} = -\Pi_T^{W\,(1)}(\hat{M}_W^2)/\hat{M}_W \sim g^2 M_W$.

Unstable particle EFT (IV)

Gauge invariance is automatic, since

- The full electroweak SM is SU(2)×U(1)$_Y$ gauge-invariant
- The effective Lagrangian is U(1)$_{\text{em}}$ gauge-invariant.
- The matching equations are formulated as "on-shell" equations at the complex pole of the W propagator (including a complex LSZ residue factor) which are (SU(2)×U(1)$_Y$) gauge-independent, e.g.

$$[\sqrt{R_e}]^2[\sqrt{R_W}]^2\,\mathcal{A}(e^-e^+\to W^-W^+)_{|p_W^2=\bar{s}} = C_i\,[\varpi^{-1/2}]^2\,\mathcal{A}_i(e^-e^+\to W^-W^+)_{\text{eff,tree}},$$

where

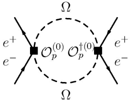
$$\varpi^{-1} \equiv \left(1+\frac{M_W\Delta+\vec{k}_i^{\,2}}{M_W^2}\right)^{1/2}$$

is a field normalization factor for non-relativistic fields (usually E/M).

Born cross section (I)

Calculation of the LO cross section

$$i\mathcal{A}^{(0)}_{LR} \quad = \quad$$



$$= \frac{\pi^2\alpha_{ew}^2}{M_W^4}\, 16 M_W^2 \int \frac{d^d r}{(2\pi)^d} \frac{1}{\left(r^0 - \frac{\vec{r}^{\,2}}{2M_W} - \frac{\Delta^{(1)}}{2}\right)\left(E - r^0 - \frac{\vec{r}^{\,2}}{2M_W} - \frac{\Delta^{(1)}}{2}\right)}$$

$$= -4i\pi\alpha_{ew}^2 \sqrt{-\frac{E + i\Gamma_W^{(0)}}{M_W}}\,.$$

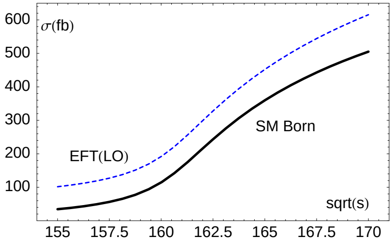
Since

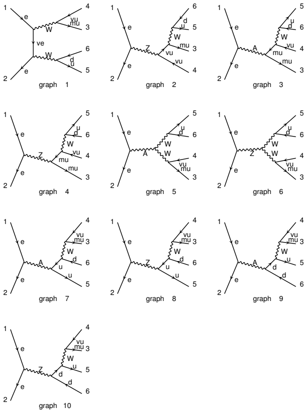
$$\mathrm{Cut}\left[\frac{1}{r^0 - \frac{\vec{r}^{\,2}}{2M_W} - \frac{\Delta^{(1)}}{2}}\right] = \frac{\Gamma_W^{(0)}}{\left|r^0 - \frac{\vec{r}^{\,2}}{2M_W} - \frac{\Delta^{(1)}}{2}\right|^2}$$

obtain the flavour-specific $\mu^-\bar{\nu}_\mu u\bar{d}$ cross section by multiplying with $\Gamma^{(0)}_{W,\mu\nu_\mu}\Gamma^{(0)}_{W,ud}/\Gamma_W^{(0)\,2} = 1/27$.

Born cross section (II)

LO is a poor approximation to the Born cross section *defined* with a fixed width prescription (computed e.g. with CompHep+Whizard)





Born cross section (III)

NLO correction from the potential region (production operators and propagator insertions)

$$i\mathcal{A} = \int \frac{d^d r}{(2\pi)^d}\, \Phi(E,r)P(k_1)P(k_2)$$

Expansion of the off-shell W pair production matrix element (around the complex "on-shell" conditi-on)

$$\Phi_{LR}(E,r) = -4g_{ew}^4 \left[1 + \left(\frac{11}{6} + 2\xi^2(s) + \frac{38}{9}\xi(s)\right)\frac{\vec{r}^{\,2}}{M_W^2}\right] + O(\delta^2)$$

with $\xi(s) = -\frac{3M_W^2(s-2M_Z^2 s_w^2)}{s(s-M_Z^2)}$ from the s-channel diagrams. The propagator corrections arise from the kinetic energy correction, the two-loop self-energy and the complex residue:

$$\begin{aligned} P(r) \quad = \quad & \frac{i}{2M_W\left(r_0 - \frac{\vec{r}^{\,2}}{2M_W} - \frac{\Delta^{[1]}}{2}\right)}\left(1 + \Pi^{(1,1)} - \frac{M_W\Delta^{[1]} + \vec{r}^{\,2}}{2M_W^2}\right) \\ & - \frac{i\left[\left(\frac{\vec{r}^{\,2}}{2M_W} + \frac{\Delta^{[1]}}{2}\right)^2 - M_W\Delta^{(2)}\right]}{4M_W^2\left(r_0 - \frac{\vec{r}^{\,2}}{2M_W} - \frac{\Delta^{[1]}}{2}\right)^2} - \frac{i}{4M_W^2} + O\left(\frac{\delta}{M_W^2}\right) \qquad (1) \end{aligned}$$

Born cross section (IV)

Hard (non-resonant) $N^{1/2}LO$ and $N^{3/2}LO$ corrections

- Contribution to the matching coefficient of

$$\mathcal{O}_{4e}^{(k)} = \frac{C_{4e}^{(k)}}{M_W^2}(\bar{e}_{c1}\Gamma_1 e_{c2})(\bar{e}_{c2}\Gamma_2 e_{c1}),$$

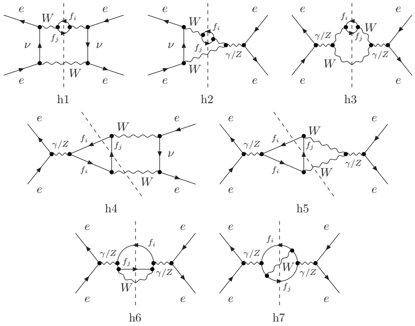
  Computed in standard PT with propagator $-ig_{\mu\nu}/(p^2 - M_W^2)$.

- Two-loop cut diagrams result in

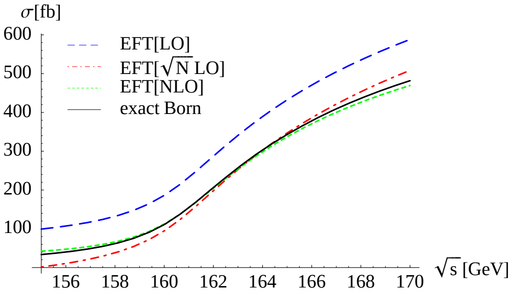
$$\sigma_{LR}^{(1/2+3/2)} = \frac{4\alpha^3}{27 s_w^6 s}\left[K_{h1} + K_{h2}\,\frac{E}{M_W}\right]$$

  Contribution from the diagrams h4, h5 and with a single W turns out to be very small.

- The leading term is $N^{1/2}LO$, because of loop suppression, but absence of threshold suppression $\propto \sqrt{\delta}$ of the potential region.
- Three-loop diagrams give another $N^{3/2}LO$ term $\propto \alpha^4$ but energy-independent.



Born cross section (V)



	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d})$(fb)				
$\sqrt{s}$ [GeV]	EFT(LO)	EFT($\sqrt{\text{N}}$LO)	EFT(NLO)	EFT($\text{N}^{\frac{3}{2}}$ LO)	exact Born
155	101.61	1.62	43.28	31.30	34.43(1)
158	135.43	39.23	67.78	62.50	63.39(2)
161	240.85	148.44	160.45	160.89	160.62(6)
164	406.8	318.1	313.5	318.8	318.3(1)
167	527.8	442.7	420.4	429.7	428.6(2)
170	615.5	533.9	492.9	505.4	505.1(2)

Radiative corrections (I)

Radiative corrections correspond to cuts involving loops on each side of the cut or five-particle $\mu^- \bar{\nu}_\mu u\bar{d}\,\gamma$ cuts. Up to NLO:

- Two-loop $\Delta^{(2)} = M_W(\Pi^{(2,0)} + \Pi^{(1,1)}\Pi^{(1,0)}) = -i\Gamma_W^{(1)}$, i.e. one-loop EW correction to on-shell W decay in the pole mass renormalization scheme.
- One-loop EW correction to the LO production operator

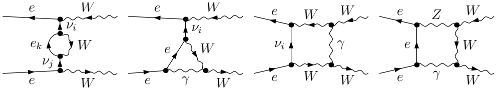
$$\mathcal{O}_p^{(1)} = \frac{\pi\alpha_{ew}}{\hat{M}_W^2}\left[C_{p,LR}^{(1)}\left(\bar{e}_L\gamma^{[i}n^{j]}e_L\right) + C_{p,RL}^{(1)}\left(\bar{e}_R\gamma^{[i}n^{j]}e_R\right)\right]\left(\Omega_-^{\dagger i}\Omega_+^{\dagger j}\right)$$

- Up to two insertions of the Coulomb potential interaction.
- Soft and collinear photon corrections to the EFT forward-scattering amplitude.
- Resummation of large collinear logarithms $\ln(M_w/m_e)$ from initial-state radiation.

Up to NLO QCD affects mainly the hadronic partial W decay width. Mixed three-loop QCD/EW hard effects are small and will be neglected.

Radiative corrections (II)

NLO matching of the leading production operator



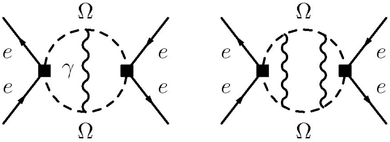
31 box, 84 vertex, 65 self-energy diagrams, but many of them vanish at LO in the expansion around $s = 4M_W^2$.

- Renormalization conventions: G_μ scheme with $M_W = 80.377\,\text{GeV}$, M_Z and G_μ as input, α_{ew}, α, $\sin\theta_W$ derived. On-shell field renormalization. $m_t = 174.2\,\text{GeV}$, $m_H = 115\,\text{GeV}$.
- $$C^{(1)}_{p,LR} = \frac{\alpha}{2\pi}\left[\left(-\frac{1}{\epsilon^2} - \frac{3}{2\epsilon}\right)\left(-\frac{4M_W^2}{\mu^2}\right)^{-\epsilon} - 10.076 + 0.205\,i\right]$$

  Imaginary part corresponds to wrong cuts and must be dropped.
- Technically the most complicated part of NLO calculation, but completely standard. Much simpler than the 1-loop hexagon diagrams in the full NLO $e^- e^+ \to 4f$ calculation.

Radiative corrections (III)

Coulomb correction

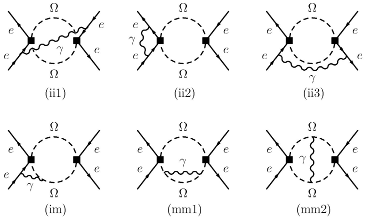


$$\Delta\sigma^{(1)}_{LR,\text{Coulomb}} = \frac{4\pi\alpha^2}{27 s_w^4 s}\,\text{Im}\left[-\frac{\alpha}{2}\ln\left(-\frac{E+i\Gamma_W^{(0)}}{M_W}\right) + \frac{\alpha^2\pi^2}{12}\sqrt{-\frac{M_W}{E+i\Gamma_W^{(0)}}}\,\right].$$

- Single Coulomb exchange is $\mathrm{N}^{1/2}$LO. About 5%, the largest radiative correction *(Fadin, Khoze, Stirling, 1993)*.
- Double Coulomb exchange is NLO. Down to a few permille.

Radiative corrections (IV)

Soft and collinear photon correction



$$\Delta\mathcal{A}^{(1),\text{fin}}_{LR,\text{soft}} = \mathcal{A}^{(0)}_{LR}\,\frac{2\alpha}{\pi}\left[\ln^2\left(-\frac{8(E+i\Gamma^{(0)}_W)}{\mu}\right) - 4\ln\left(-\frac{8(E+i\Gamma^{(0)}_W)}{\mu}\right) + 8 + \frac{13}{24}\pi^2\right]$$

- Poles have been subtracted. Double pole cancels with one-loop correction to production operator. Single pole must be subtracted into the electron distribution function.
- Cancellation of (mm) and (im) diagrams *(Fadin, Khoze, Martin, 1994; Melnikov, Yakovlev, 1994)* is trivial in the EFT, since the soft photon coupling to the W fields, $\Omega^{\dagger i}_{\mp} A^0_s \Omega^i_{\mp}$, can be gauged away. (ii2) and (ii3) are scaleless.
- Collinear loops are scaleless for $m_e = 0$

Initial state radiation (I)

Sum of all radiative corrections gives the "short-distance cross section" with IR divergences due to ISR collinear singularities regulated dimensionally. The physical cross section is

$$\sigma_h(s) = \int_0^1 dx_1 \int_0^1 dx_2\, \Gamma_{ee}^{\overline{\mathrm{MS}}}(x_1)\Gamma_{ee}^{\overline{\mathrm{MS}}}(x_2)\, \hat{\sigma}_h^{\overline{\mathrm{MS}}}(x_1 x_2 s)$$

We convert this to the standard scheme, where these singularities are regulated by the (physical) electron mass: $\hat{\sigma}_h^{\overline{\mathrm{MS}}}(s) \to \hat{\sigma}_h^{\mathrm{conv}}(s)$, $\Gamma_{ee}^{\overline{\mathrm{MS}}}(x) \to \Gamma_{ee}^{\mathrm{LL}}(x)$.

The conversion implies the calculation of hard-collinear and soft-collinear photon loop diagrams (electron distribution at large x), where

$$\begin{aligned} \text{hard-collinear}: &\quad k_0 \sim M_W,\; k^2 \sim m_e^2 \\ \text{soft-collinear}: &\quad k_0 \sim \Gamma_W,\; k^2 \sim m_e^2 \Gamma_W / M_W \end{aligned}$$

$$\begin{aligned} \sigma_{LR}^{(1)}(s) &= \frac{1}{27s}\mathrm{Im}\left[\mathcal{A}_{LR}^{(0)}\frac{\alpha}{\pi}\left(4\ln\left(-\frac{4(E+i\Gamma_W)}{M_W}\right)\ln\left(\frac{2M_W}{m_e}\right) - 5\ln\left(\frac{2M_W}{m_e}\right)\right.\right. \\ &\quad \left.\left. +\mathrm{Re}\left[C_{p,LR}^{(1,\mathrm{fin})}\right] + \frac{\pi^2}{4} + 3\right)\right] + \Delta\sigma_{LR,\mathrm{Coulomb}}^{(1)} + \Delta\sigma_{LR,\mathrm{decay}}^{(1)}\,. \end{aligned}$$

Initial state radiation (II)

Contains large logarithms $\alpha \ln(2M_W/m_e)$.
To sum them to all orders, write $\Gamma_{ee}^{\rm LL}(x) = \delta(1-x) + \Gamma_{ee}^{{\rm LL},(1)}(x) + O(\alpha^2)$, and calculate

$$\begin{aligned}
\hat{\sigma}^{(1)}_{LR,{\rm conv}}(s) &= \sigma^{(1)}_{LR}(s) - 2\int_0^1 dx\, \Gamma_{ee}^{{\rm LL},(1)}(x)\, \sigma^{(0)}_{LR,{\rm Born}}(xs) \\
&= \frac{4\alpha^3}{27 s_w^4 s}\,{\rm Im}\left\{(-1)\sqrt{-\frac{E+i\Gamma_W^{(0)}}{M_W}}\left(2\ln\left(-\frac{4(E+i\Gamma_W^{(0)})}{M_W}\right)\right.\right. \\
&\quad \left.\left. + {\rm Re}\left[C_{p,LR}^{(1,{\rm fin})}\right] + \frac{\pi^2}{4} + \frac{1}{2}\right)\right\} + \Delta\sigma^{(1)}_{\rm LR,Coulomb} + \Delta\sigma^{(1)}_{\rm LR,decay}\,.
\end{aligned}$$

This is free from large logs of m_e, but it contains logs of Γ/M which could be summed similar to the summation of $\ln(1-x)$ in DIS or DY in QCD. Final result is obtained from

$$\sigma_h(s) = \int_0^1 dx_1 \int_0^1 dx_2\, \Gamma_{ee}^{\rm LL}(x_1)\Gamma_{ee}^{\rm LL}(x_2)\hat{\sigma}_h^{\rm conv}(x_1 x_2 s)$$

with electron structure functions as in the LEP2 Yellow Book.

NLO Result (I)

Born, ISR-improved Born and NLO calculation in the EFT (with two implementation of ISR: convolution of the NLO partonic cross section, or Born cross section only)

	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d} X)$(fb)			
$\sqrt{s}$ [GeV]	Born (SM)	Born-ISR (SM)	EFT(ISR-NLO)	EFT(ISR-Tree)
158	61.67(2)	45.64(2) [-26.0%]	49.19(2) [-20.2%]	50.02(2) [-18.9%]
161	154.19(6)	108.60(5) [-29.6%]	117.81(5) [-23.6%]	120.00(5) [-22.2%]
164	303.0(1)	219.7(1) [-27.5%]	234.9(1) [-22.5%]	236.8(1) [-21.8%]
167	408.8(2)	310.2(1) [-24.1%]	328.2(1) [-19.7%]	329.1(1) [-19.5%]
170	481.7(2)	378.4(2) [-21.4%]	398.0(2) [-17.4%]	398.3(2) [-17.3%]

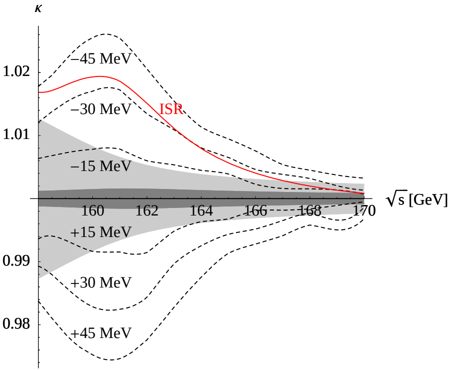
Comparison with of Born, EFT, full four fermion (*Denner, Dittmaier, Roth, Wieders, 2005*) and DPA NLO calculations, ISR resummed. Same input parameters.

	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d} X)$(fb)			
$\sqrt{s}$ [GeV]	Born (SM)	EFT	full ee4f	DPA
161	107.06(4)	117.38(4)	118.12(8)	115.48(7)
170	381.0(2)	399.9(2)	401.8(2)	402.1(2)

NLO Result (II)

Sensitivity to M_W and theoretical uncertainty
Variation of cross section normalized to standard input

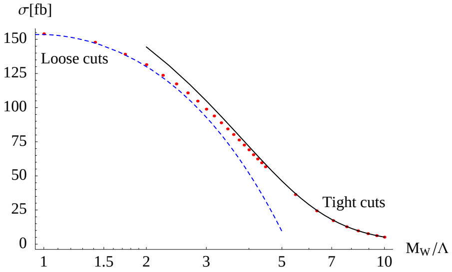
- At the point of maximal sensitivity large uncertainty from current implementation of ISR ($\delta M_W \approx 30\,\text{MeV}$)
- Uncertainties from $N^{3/2}$LO radiative effects are estimated 10 MeV from hard corrections and 4 MeV from Coulomb times hard + soft
- Experimental accuracy (6 MeV) can be reached by NLL ISR implementation and inclusion of $N^{3/2}$LO – use existing full NLO $4f$ calculation plus dominant NNLO terms from EFT approach.
- Probably also need a less inclusive treatment.



EFT and cuts (I)

Cuts are not straightforward in the EFT approach: may introduce new scales regions.
Example: Invariant mass cuts $|M^2_{u\bar{d}} - M^2_W|, |M^2_{\mu\bar{\nu}_\mu} - M^2_W| < \Lambda^2$

- Loose cut: $\Lambda \sim M_W$
  No modification of potential loops (momenta always within the cut by power counting).
  Cut affects the matching coefficient of the four-electron operator (non-resonant terms).
- Tight cut: $\Lambda \sim \sqrt{M_W \Gamma_W}$
  Four-electron operator (non-resonant terms) does not contribute at all.
  Cut affects loop calculations in the effective theory.



Red dots: Born cross section ($\sqrt{s} = 161$ GeV, WHIZARD)

EFT and cuts (II)

W mass measurement uses almost the inclusive cross section. Cut for the cross section measurement at $\sqrt{s} = 161$ GeV used at LEP:

Cut	$\sigma_{\text{Born}}(e^- e^+ \to \mu^- \overline{\nu}_\mu u\bar{d})$(fb)	$\sigma_{\text{cut}} / \sigma_{\text{tot}}$
–	154.18(5)	
$\lvert\vec{p}_\mu\rvert > 20$ GeV	153.71(5)	99.69(5) %
$M_{\mu\nu} > 55$ GeV, 40 GeV $< M_{jj} < 120$ GeV	150.61(5)	97.68(5) %
$\theta_{\mu j} > 15$ degrees	149.35(5)	96.87(5) %
$\lvert\cos\theta_\nu\rvert < 0.95$	148.28(5)	96.17(5) %
all	140.03(5)	90.82(5) %

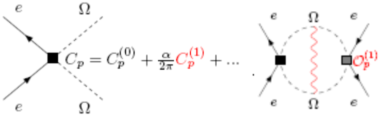
2nd and 3rd column: Effect of LEP phase-space cuts on the Born cross section at $\sqrt{s} = 161$ GeV

Strategy: Use full NLO computation à la Denner-Dittmaier (complex mass scheme) including all cuts + EFT calculation of the leading NNLO terms without cuts ($\approx 7\%$ error on a small correction).

Beyond NLO (I)

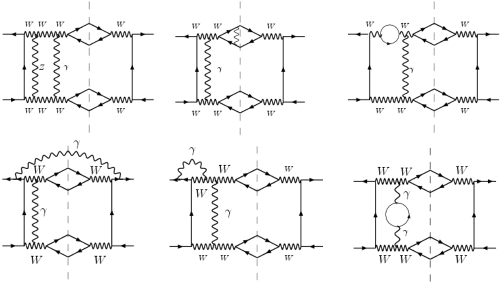
Dominant NNLO = $N^{3/2}$LO in EFT counting

- NLO correction to non-resosnant four-electron operator – already included in full NLO (non-resonant Born terms were $N^{1/2}$L0).
- Interference of Coulomb exchange with tree-level higher-dimensional production operators – already included in full NLO.

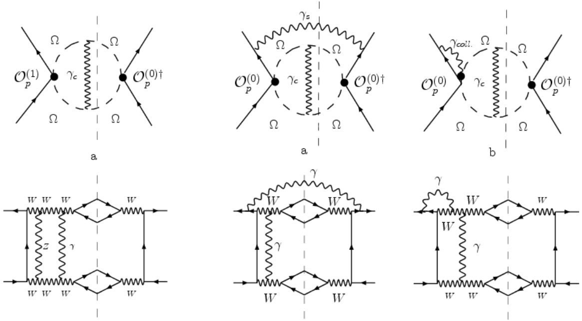


$N^{3/2}$LO terms from true NNLO diagrams contain at least one Coulomb photon:

- Mixed hard/Coulomb corrections
- Interference of Coulomb exchange with soft and collinear radiative corrections
- Correction to the Coulomb potential itself.



Beyond NLO (II)



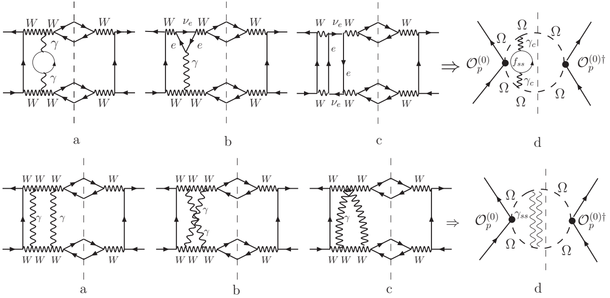
Result for [hard+soft+collinear]×Coulomb, to be convoluted with electron structure functions:

$$\hat{\sigma}^{C}_{LR}(s) = \frac{16\pi\alpha_{ew}^2\alpha}{27 s M_W^2}\left[\left(\frac{9}{2}+\frac{\pi^2}{4}+\mathrm{Re}\, c^{(1),\mathrm{fin}}_{p,LR}\right)\mathrm{Im}\, G^{(0)}_{\mathrm{C}}(0,0;\mathcal{E}_W) + 2\,\mathrm{Im}\int_0^\infty dk\, \frac{G^{(0)}_{\mathrm{C}}(0,0;\mathcal{E}_W - k)}{[k]_{M_W+}}\right]$$

$$\longrightarrow -\frac{\alpha_{ew}^2\,\alpha^2}{27\,s}\left\{\left(9+\frac{\pi^2}{2}+2\,\mathrm{Re}\, c^{(1),\mathrm{fin}}_{p,LR}\right)\mathrm{Im}\left[\ln\left(-\frac{\mathcal{E}_W}{M_W}\right)\right] + 2\,\mathrm{Im}\left[\ln^2\left(-\frac{\mathcal{E}_W}{M_W}\right)\right]\right\}.$$

Beyond NLO (III)

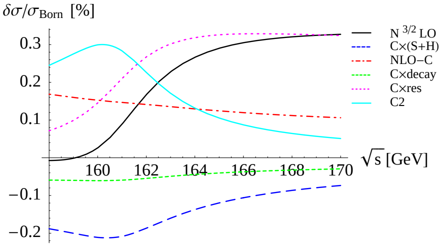
NLO correction to the Coulomb potential



$$\Delta\sigma_{LR}^{\mathrm{NLO-C}} = -\frac{\alpha_{ew}^2\alpha^2}{81s}\sum_f C_f Q_f^2\left\{4\ln\left(\frac{2M_W}{M_Z}\right)\mathrm{Im}\left[\ln\left(-\frac{\mathcal{E}_W}{M_W}\right)\right]+\mathrm{Im}\left[\ln^2\left(-\frac{\mathcal{E}_W}{M_W}\right)\right]\right\}$$

$$+\,\delta_{\alpha(M_Z)\to G_\mu}\,\Delta\sigma_{LR}^{\mathrm{C1}}$$

Beyond NLO (IV)



In total a small correction:

$[\delta M_W]_{\rm BeyondNLO} \approx (3-5)\,\text{MeV}$

	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d} X)$(fb)				
$\sqrt{s}$ [GeV]	Born	Born (ISR)	NLO	$\hat{\sigma}^{(3/2)}$	$\sigma^{(3/2)}_{\rm ISR}$
158	61.67(2)	45.64(2) [−26.0%]	49.19(2) [−20.2%]	−0.001 [−0.00%]	0.000 [+0.00%]
161	154.19(6)	108.60(4) [−29.6%]	117.81(5) [−23.6%]	0.147 [+0.10%]	0.087 [+0.06%]
164	303.0(1)	219.7(1) [−27.5%]	234.9(1) [−22.5%]	0.811 [+0.27%]	0.544 [+0.18%]
167	408.8(2)	310.2(1) [−24.1%]	328.2(1) [−19.7%]	1.287 [+0.31%]	0.936 [+0.23%]
170	481.7(2)	378.4(2) [−21.4%]	398.0(2) [−17.4%]	1.577 [+0.33%]	1.207 [+0.25%]

Summary

- First dedicated theoretical study of the W pair production threshold region including finite width effects.
- Also first application of unstable particle effective field theory to a Standard Model process.
- Experimental accuracy can be matched by combining:
  - Full SM NLO calculation (or a corresponding EFT calculation)
  - Inclusion of dominant NNLO ($N^{3/2}LO$) effects in the EFT framework.
  - Improved treatment of collinear logarithms at NLL through electron distribution functions in analogy with PDFs in QCD – common to ILC precision physics.
- Future work on unstable particle EFT should focus on distributions and implementation of cuts.