QCD scattering amplitudes beyond Feynman diagrams

MHV, CSW, BCFW and all that

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Multi-particle final states important for LHC:

- Higgs in Vector-Boson fusion: $VV + jj$
- Higgs +top production: $t\bar{t}H \to t\bar{t}b\bar{b} \quad \Leftrightarrow \quad t\bar{t}jj$
- SUSY signals $\bar{q}q\bar{q}q + \chi^0\chi^0 \Leftrightarrow \quad 4j + Z \to 4j + \nu\bar{\nu}$

Rapid growth of # of Feynman diagrams:

$2 \to 2$ gluon tree amplitude: 4 diagrams

...

$2 \to 6$ gluon tree amplitude: 34300 diagrams

$\Rightarrow$ Efficient methods needed

- Color decomposition, spinor methods
- Recursive methods, SUSY-relations, unitarity methods...
- Closed expression for "maximally helicity violating" amplitudes

Since 2003: New methods (mostly) for massless QCD amplitudes

- Relation to twistor string theory (Witten 2003)

⇒ New representations of QCD amplitudes

- CSW rules (Cachazo, Svrček, Witten 04)
  - All massless born QCD amplitudes from MHV vertices
  - Loop diagrams in SUSY theories (Brandhuber, Spence, Travaglini 04)
- BCFW rules: (Britto, Cachazo, Feng/Witten, 04/05)
  - Construct born amplitudes from on-shell subamplitudes
  - Rational part of one loop amplitudes (Bern, Dixon, Kosower 05)
- Unitarity methods (Britto, Cachazo, Feng 04, Anastasiou et.al 06, Forde 07,...)

Common ideas:

on-shell amplitudes as building blocks, complex kinmatics

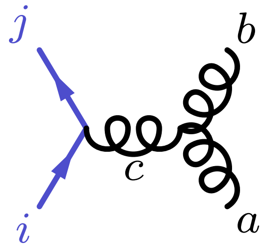
Generalization of new methods to massive particles?

- **Theoretical questions:** CSW/BCFW representations
  - properties of QFT or specific to (unbroken) SUSY, QCD...?
- **LHC phenomenology**

Overview

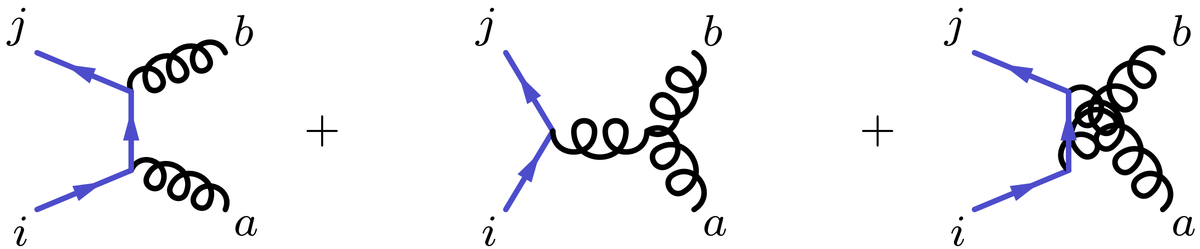
- **N-Gluon amplitudes** Color decomposition, Helicity methods,
  - MHV amplitudes, Berends-Giele recursion
- **MHV diagrams**
  - Extension to massive scalars (R.Boels, CS 07)
- **SUSY relations** for massive quarks and scalars (CS, S.Weinzierl 06)
- **BCFW recursion**
  - Extension to massive scalars and quarks
  - (Badger et.al 05, CS, S.Weinzierl, 07)

Simplifying color factors using Lie Algebra $[T^a, T^b] = \mathrm{i} f^{abc} T^c$



$$\propto \quad \begin{aligned} & -\mathrm{i} T^c_{ij} f^{abc} V^{\mu_a \mu_b \mu_c}_{ggg} \\ & = (\mathrm{i})^2 \left[(T^a T^b)_{ij} V^{\mu_a \mu_b \mu_c}_{ggg} + (T^b T^a)_{ij} V^{\mu_b \mu_a \mu_c}_{ggg} \right] \end{aligned}$$

Color decomposition: into color ordered partial amplitudes



$(T^a T^b)_{ij}$ $\qquad$ $(T^a T^b)_{ij} + (T^b T^a)_{ij}$ $\qquad$ $(T^b T^a)_{ij}$

$$\mathcal{A}_4(i_Q, a, b, j_{\bar{Q}}) = (T^a T^b)_{ij} A_4(i_Q, a, b, j_Q) + (T^b T^a)_{ij} A_4(i_Q, b, a, j_Q)$$

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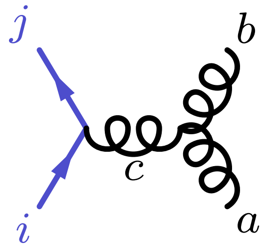

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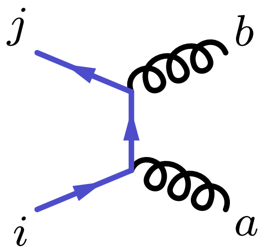
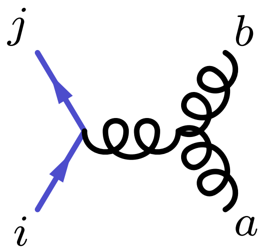


$(T^a T^b)_{ij}$

$+$



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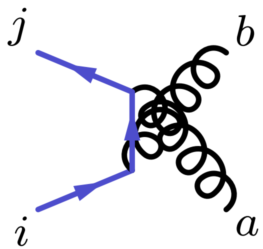

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General decomposition (Berends,Giele, 1987):

$$\mathcal{A}_{n+2}(i_Q, 1, 2, ..., n, j_{\bar{Q}}) = g^n \sum_{\sigma \in S_n} (T^{a_{\sigma(1)}} ... T^{a_{\sigma(n)}})_{i,j} \, A_n \left(i_Q, \sigma(1), ..., \sigma(n), j_{\bar{Q}} \right)$$

Two-component Weyl spinors in braket notation

$$|k+\rangle = \lambda_{k,A} = \left(\tfrac{1+\gamma^5}{2}\right) u(k) \quad , \quad |k-\rangle = \bar{\lambda}_k^{\dot{A}} = \left(\tfrac{1-\gamma^5}{2}\right) u(k)$$

- Express momenta in terms of spinors: $\langle k + |\gamma^\mu| k+\rangle = 2k^\mu$
- antisymmetric spinor products $\langle pk\rangle = \langle p - |k+\rangle$, $[pk] = \langle p + |k-\rangle$

Polarization vectors of the external gluons

(Kleiss,Sterling; Gunion, Kunszt 1985, Xu, Zhang, Chang 1987)

$$\epsilon_\mu^\pm(k,q) = \pm \frac{\langle q \mp |\gamma_\mu| k\mp\rangle}{\sqrt{2}\,\langle q \mp | k\pm\rangle}$$

with q arbitrary light-like reference momentum

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Closed expression for MHV amplitudes (Parke-Taylor 1986)

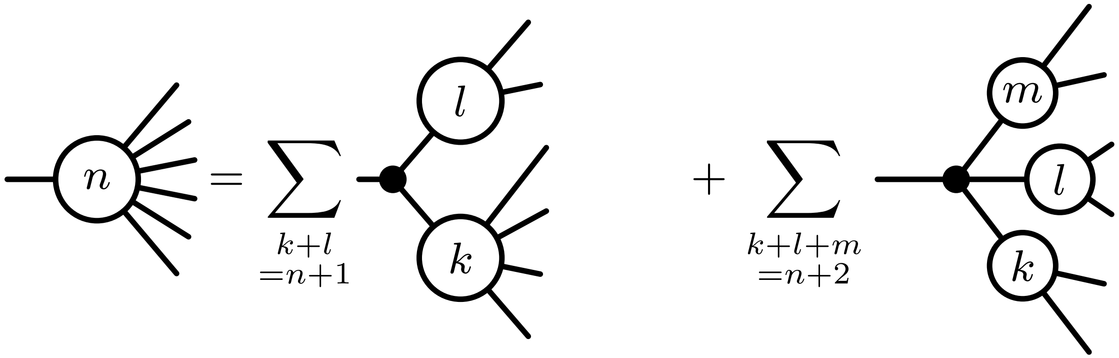
$$A_n(g_1^+,\ldots,g_i^-,\ldots,g_j^-,\ldots g_n^+) = \mathrm{i}2^{n/2-1}\frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

Traditional proof of MHV-formula: (Berends, Giele 1988)

Recursion relations for one particle off-shell tree amplitudes

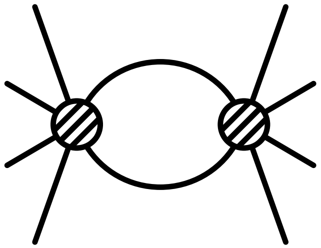
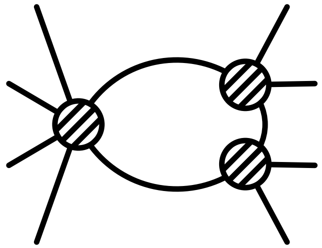
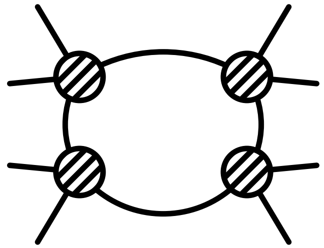
$$A_n(1,\ldots,(n-1),\widehat{n}) = \langle 0|\phi_n(k_n)|k_1,\ldots k_{n-1}\rangle$$

Can be constructed recursively:



- Avoids redundancies ⇒ useful for numerical calculations.
  `Alpha` (Caravaglios, Moretti),
  `Helac` (Kanaki, Papadopoulos), `O'Mega` (Moretti, Ohl, Reuter, CS)
- *But*: off-shell amplitudes required, not ideal for analytical calculations

Decompostion of one-loop amplitudes into scalar integrals

$\mathcal{A} = \sum c_i^{(2)}$  $+ \; c_{i,j}^{(3)}$  $+ \; c_{i,j,k}^{(4)}$  $+ \; \mathcal{R}$

$B_0(k_{1,i}, k_{i,n})$ $\quad C_0(k_{1,i}, k_{i,j}, k_{j,n})$ $\quad D_0(k_{1,i}, k_{i,j}, k_{j,k}, k_{k,n})$

with

- $c^{(n)}$: independent of ϵ
- $\mathcal{R}$: "rational part" from $\epsilon \times \frac{1}{\epsilon}$

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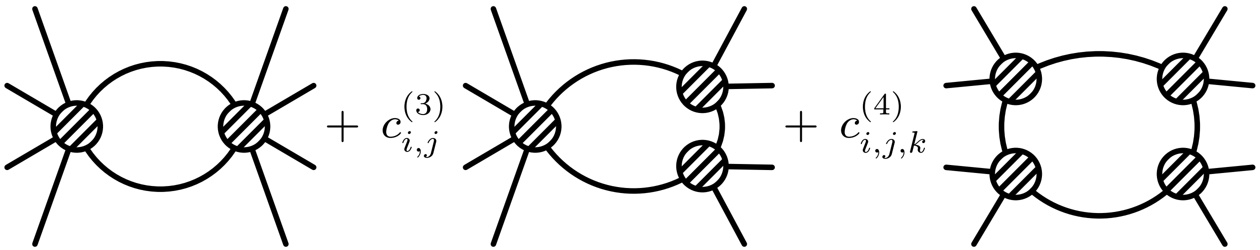

$$\mathcal{A} = \sum c_i^{(2)} \, B_0(k_{1,i}, k_{i,n}) + c_{i,j}^{(3)} \, C_0(k_{1,i}, k_{i,j}, k_{j,n}) + c_{i,j,k}^{(4)} \, D_0(k_{1,i}, k_{i,j}, k_{j,k}, k_{k,n}) + \mathcal{R}$$

with

- $c^{(n)}$: independent of ϵ
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Unitarity method: reconstruct amplitude from imaginary part

- evaluate cuts in four dimensions:
  "cut-constructable part" (miss R) (Bern, Dixon, Dunbar, Kosower 94)
- evaluate cuts in D-dimensions (Bern, Morgan 95)

So far: All-multiplicity solution for MHV amplitudes

Useful for all amplitudes?

Earlier attempts

- MHV amplitudes from 2-D field theory (Nair 88)
- relation to self-dual Yang-Mills (Bardeen; Cangemi; Chalmers, Siegel 96)

Insights from Twistor space (Witten 2003)

- "Half a Fourier transform":

$$(\lambda_A, \bar{\lambda}^{\dot{A}}) \Rightarrow (\lambda_A, \mu^{\dot{A}}) = (\lambda_A, \frac{\partial}{\partial \bar{\lambda}_{\dot{A}}})$$

- MHV amplitudes nonvanishing on line in twistor space
- Conjectures
  - All QCD amplitudes lie on curves in twistor space, determined by # of negative helicities and loops
  - Can be computed in string theory on twistor space

$\Rightarrow$ New representations of QCD amplitudes

MHV diagrams (CSW rules): (Cachazo, Svrček, Witten 2004)

All QCD amplitudes from MHV vertices

$$V_{\text{CSW}}(1^+ \ldots i^- \ldots j^- \ldots n^+) = 2^n \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle}$$

with off-shell continuation $|k+\rangle \to \not{k}\,|\eta-\rangle$

Scalar propagators $\frac{\mathrm{i}}{k^2}$ connecting $+$ and $-$ labels

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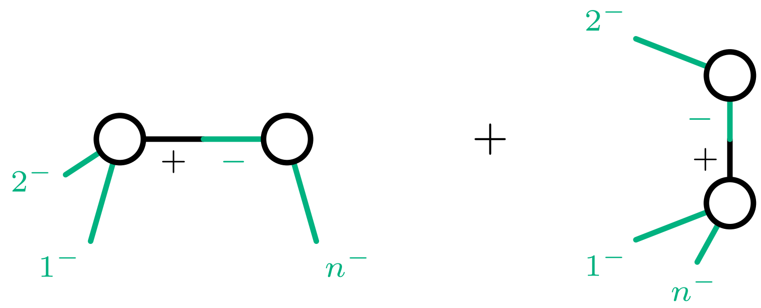
Example: NMHV amplitudes $A(1^-, 2^-, 3^+, \dots n^-)$:

- Distribute negative helicities over $d = n^- - 1 = 2$ MHV vertices



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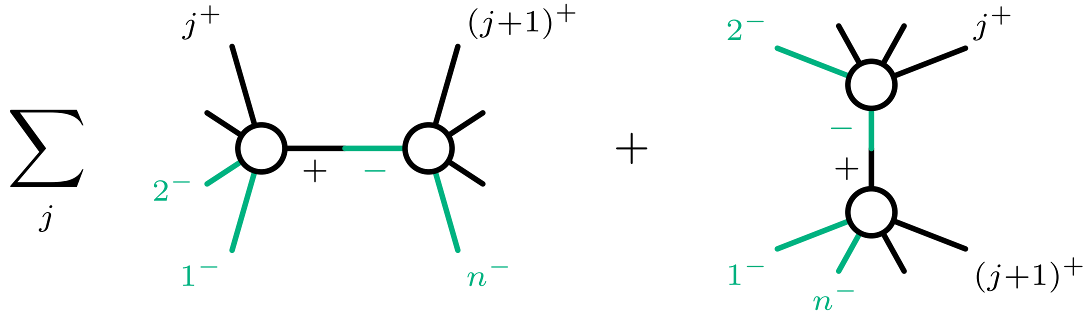

- Distribute positive helicities $\Rightarrow 2(n-3)$ diagrams

Color ordered amplitudes

(Dindsdale, Ternik, Weinzierl, 06)

n	5	6	7	8	9	10
BG	0.23 ms	0.9	3	11	30	90
CSW*	0.4	4.2	33	240	1770	13000

Averaged color ordered/dressed amplitudes

(Duhr, Höche, Maltoni, 06)

n	5	6	7	8	9	10
BG (CD)	0.27 ms	0.72	2.37	8.21	27	86.4
BG (CO)	0.38	1.42	5.9	27.6	145	796
CSW (CO)†	0.6	2.78	14.6	91.9	631	4890

Cross sections

(SHERPA,Gleisberg et.al. 07)

n	5(g)	6(g)	6(j)	7(g)	7(j)
Feynman	1.4 ks	90	210	-	-
CSW	0.2	0.6	5.8	17	122

(*: recursive reformulation (Bena, Bern, Kosower, 04); †: reformulation with cubic vertices)

Massive particles: External Higgs or gauge bosons

(Dixon, Glover ,Khoze 04; Bern, Forde, Kosower, Mastrolia 04)

Loop diagrams in SUSY theories, (Brandhuber, Spence, Travaglini 04)

"cut constructable" part of QCD amplitudes

Derivations:

- Generalized BCFW recursion (Risager 05)
- Field-redefinition in light-cone QCD (Mansfield 05)
- Yang-Mills theory on twistor space

  (Boels, Mason, Skinner 06)

⇒ Proposals for one loop CSW rules in QCD

(Ettle,Fu, Fudger, Mansfield, Morris; Brandhuber, Spence, Travaglini, Zoubos, 07)

⇒ Use to derive CSW rules for propagating massive scalars

(R.Boels, CS, in preparation)

- Light-cone decomposition

$$A_{\pm} = \frac{1}{\sqrt{2}}(A_0 \mp A_3), \qquad A_{z/\bar{z}} = \frac{1}{\sqrt{2}}(-A_1 \pm \mathrm{i}A_2)$$

impose light-cone gauge $A_+ = \nu \cdot A = 0$ with $\nu \sim (1,0,0,1)$,

- eliminate A_- by e.o.m $\Rightarrow$ Lagrangian for physical fields A_z, $A_{\bar{z}}$:

$$\mathcal{L}^{(2)} + \mathcal{L}^{(3)}_{++-} + \mathcal{L}^{(3)}_{+--} + \mathcal{L}^{(4)}_{++--}$$

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- Canonical transformation $A_z \to B[A_z]$ (Mansfield 05)
eliminates $\mathcal{L}^{(3)}_{++-}$ and generates MHV-type vertices:

$$\mathcal{L}^{(3)}_{++-} + \mathcal{L}^{(3)}_{+--} + \mathcal{L}^{(4)}_{++--} \Rightarrow \sum_n \mathcal{L}^{(n)}_{+\cdots+--}$$

- Explicit solution (Ettle,Morris 06)

$$A_z(p) = \sum_{n=1}^{\infty} \int \prod_{i=1}^{\infty} \widetilde{dk_i}\, \frac{(g\sqrt{2})^{n-1} \langle \nu p \rangle^2}{\langle \nu 1 \rangle \langle 12 \rangle \ldots \langle (n-1) n \rangle \langle \nu n \rangle} B(k_1) \ldots B(k_n)$$

Similar solution for $A_{\bar{z}} \sim \sum_n B_1 \ldots \bar{B} \ldots B_n$

Application to massive scalars (R. Boels, CS, in progress)

- Lagrangian in light-cone gauge

$$\mathcal{L}^{(2)}(\bar{\phi}\phi) + \mathcal{L}^{(3)}(\bar{\phi}A_z\phi) + \mathcal{L}^{(3)}(\bar{\phi}A_{\bar{z}}\phi) + \mathcal{L}^{(4)}(\bar{\phi}A_z A_{\bar{z}}\phi) + \mathcal{L}^{(4)}(\bar{\phi}\phi\bar{\phi}\phi)$$

- eliminate $\mathcal{L}^{(3)}(\bar{\phi}A_z\phi)$ by transformation for massless scalars

$$\phi(p) = \sum_{n=1}^{\infty} \int \prod_{i=1}^{n} \widetilde{dk_i} \, \frac{(g\sqrt{2})^{n-1} \langle \nu n \rangle}{\langle \nu 1 \rangle \langle 12 \rangle \ldots \langle (n-1) n \rangle} B(k_1) \ldots B(k_{n-1}) \xi(k_n)$$

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- but mass term not invariant:

$$-m^2 \bar{\phi}(p)\phi(-p) = \sum_{n=2}^{\infty} \int \prod_{i=1}^{n} \widetilde{dp_i} \mathcal{V}_{1,\ldots,n} \bar{\xi}(k_1) B(k_2) \ldots B(k_{n-1}) \xi(k_n)$$

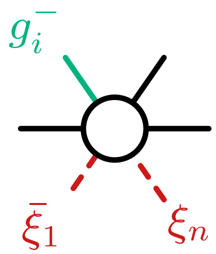
$\Rightarrow$ new CSW-vertex $\quad \mathcal{V}_{1,\ldots,n} = (g\sqrt{2})^{n-2} \dfrac{-m^2 \langle 1n \rangle}{\langle 12 \rangle \ldots \langle (n-1)n \rangle}$

Same result using Twistor Yang-Mills approach

Summary of vertices:

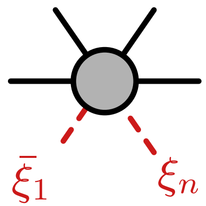
(four-scalar g^+ vertex not shown)

massless MHV vertices



$$= \mathrm{i}2^{n/2-1} \frac{\langle in\rangle^2 \langle 1i\rangle^2}{\langle 12\rangle \ldots \langle (n-1)n\rangle \langle n1\rangle}$$

holomorphic vertex $\sim m^2$



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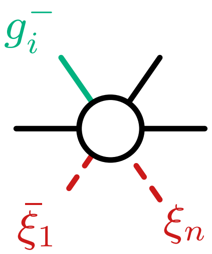

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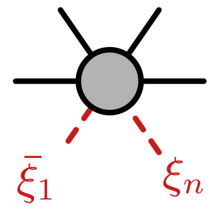

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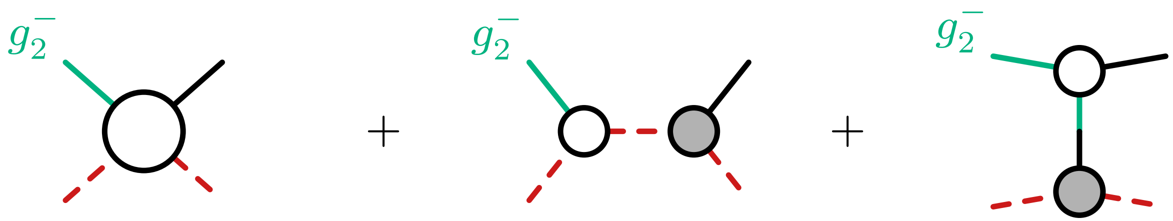
Example: $A_4(\bar{\xi}_1, g_2^-, g_3^+, \xi_4)$ (setting $|\eta+\rangle = |3+\rangle$)



$$= 2\mathrm{i}\frac{\langle 12\rangle^2\langle 24\rangle^2}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} + \frac{\sqrt{2}\mathrm{i}\langle 12\rangle\langle 2k_{1,2}\rangle}{\langle 1k_{1,2}\rangle}\frac{\mathrm{i}}{k_{1,2}^2-m^2}\frac{-\sqrt{2}\mathrm{i}m^2\langle k_{1,2}4\rangle}{\langle k_{1,2}3\rangle\langle 34\rangle}$$

CSW rules for massive scalars

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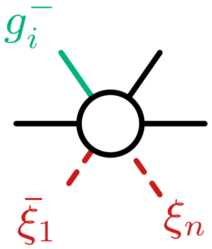
massless MHV vertices



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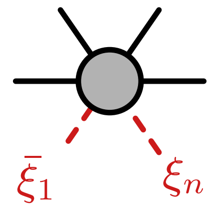

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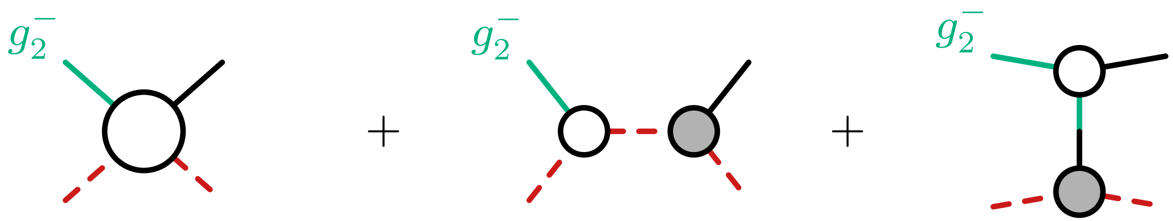

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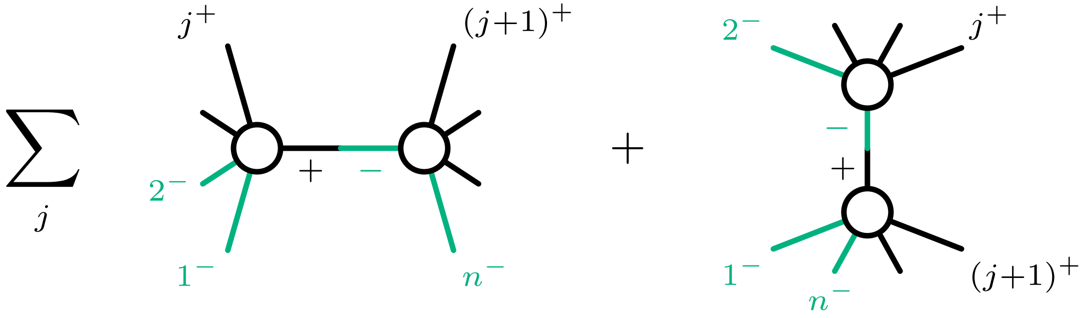
$$= 2\mathrm{i}\frac{\langle 3+|\not{k}_1|2+\rangle}{\langle 3-|\not{k}_4|3-\rangle}\frac{\langle 2-|\not{k}_4|3-\rangle^2}{\langle 23\rangle\langle 3+|\not{k}_4\not{k}_1|3-\rangle} = 2\mathrm{i}\frac{\langle 3+|\not{k}_1|2+\rangle^2}{2(k_3\cdot k_4)\langle 23\rangle[23]}$$

CSW -Summary



- construct all QCD tree-amplitudes from MHV vertices
- Lagrangian methods of derivation
- CSW rules for massive scalars
  - Massless MHV vertices + new vertex from mass term
  - Derivations should extend to massive particles with spin

Spinors for massive quarks (Kleiss, Stirling 85;. . . ; CS, S.Weinzierl 05)

$$u(\pm, q) = \frac{1}{\langle p^\flat \pm |q\mp\rangle} (\not{p} + m) |q\mp\rangle$$

with light cone projection

$$p^\flat = p - \frac{p^2}{2p \cdot q} q$$

Eigenstates of projectors $(1 \pm \not{s}\gamma^5)$ with spin vector

$$s^\mu = \frac{p^\mu}{m} - \frac{m}{(p \cdot q)} q^\mu$$

“Helicity” amplitudes depend on q!

Transformation between different reference spinors:

$$u(+, \tilde{q}) = \frac{\langle \tilde{q} - |\not{p}|q-\rangle}{\langle \tilde{q}\tilde{p}^\flat\rangle \, [p^\flat q]} u(+, q) + \frac{m \, \langle \tilde{q}q\rangle}{\langle \tilde{q}\tilde{p}^\flat\rangle \, \langle p^\flat q\rangle} u(-, q)$$

$\Rightarrow$ Need amplitudes for all quark helicities with the same $|q\pm\rangle$.

"Effective Supersymmetry" of QCD: (Parke, Taylor 1985; Kunszt 1986)

Tree level partial amplitudes for massless quarks are the same as for gluinos in a fictitious, unbroken, SUSY QCD.

SUSY transformations of helicity states of gluons and gluinos with Grassmann-valued spinor η:

$$\delta_\eta g^\pm(k) = \langle \eta \pm | k \mp \rangle \, \lambda^\pm(k) \qquad \delta_\eta \lambda^\pm(k) = -\langle \eta \mp | k \pm \rangle \, g^\pm(k)$$

SUSY Ward-Identities (Grisaru, Pendleton 1977)

$$0 = \langle 0 | [Q_{\mathsf{SUSY}}, \psi_1 \dots \psi_n] | 0 \rangle = \sum_i A_n(\psi_1 \dots (\delta_\eta \psi_i) \dots \psi_n)$$

Fermionic MHV amplitudes (Parke, Taylor 1985; Kunszt 1986)

$$A_n(\bar{\lambda}_1^-, g_2^+, \dots g_j^-, \dots, \lambda_n^+) = \frac{\langle nj \rangle}{\langle 1j \rangle} A_n(g_1^-, g_2^+, \dots g_j^-, \dots, g_n^+)$$

(set $|\eta + \rangle \propto |j+\rangle$)

Toy model: Embed QCD with massive quark $Q = \binom{\psi_+}{\psi_-}$ in $N = 1$ SQCD with two chiral Supermultiplets

$$\Psi_+ = (\varphi_+, \psi_+, F_+) \qquad , \qquad \overline{\Psi}_- = (\overline{\varphi}_-, \overline{\psi}_-, \overline{F}_-)$$

and Superpotential $W(\Psi_-, \Psi_+) = m\Psi_-\Psi_+$

SUSY Transformations of component fields

$$\delta_\eta \overline{\varphi}_- = \sqrt{2}\bar{\eta}\left(\frac{1-\gamma^5}{2}\right) Q \qquad \delta_\eta \varphi_+ = \sqrt{2}\bar{\eta}\left(\frac{1+\gamma^5}{2}\right) Q$$

$$\delta_\eta Q = -\sqrt{2}(\mathrm{i}\partial\!\!\!/ + m)\left[\varphi_+\left(\frac{1+\gamma^5}{2}\right) + \overline{\varphi}_-\left(\frac{1-\gamma^5}{2}\right)\right]\eta$$

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Transformations of helicity states $\quad ((\bar{\phi}_\pm)^\dagger = \phi_\mp)$ $\quad$ (CS, S.Weinzierl, 06)

$$\delta_\eta \phi^- = [\eta k] Q^- + m\frac{[q\eta]}{[qk]} Q^+ \qquad \delta_\eta \phi^+ = \langle \eta k\rangle\, Q^+ + m\frac{\langle q\eta\rangle}{\langle qk\rangle} Q^-$$

$$\delta_\eta Q^+ = [k\eta]\phi^+ + m\frac{\langle q\eta\rangle}{\langle qk\rangle}\phi^- \qquad \delta_\eta Q^- = \langle k\eta\rangle\, \phi^- + m\frac{[q\eta]}{[qk]}\phi^+$$

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$$\delta_q Q^+ = [kq]\phi^+ \qquad \delta_q Q^- = \langle kq\rangle\, \phi^-$$

Simplify for $|\eta\pm\rangle \propto |q\pm\rangle$ $\Rightarrow$ similar to massless case!

Only positive helicity gluons:

Quark amplitude given by scalar amplitude

$$\langle 1q\rangle\, A_n(\bar{Q}_1^+,\ldots,g_{n-1}^+,Q_n^-) = \langle nq\rangle\, A_n(\bar{\phi}_1^+,\ldots,g_{n-1}^+,\phi_n^-)$$

(SYM Lagrangian $\Rightarrow$ no $\bar{\phi}^+\bar{\lambda}^+Q$ vertex)

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Compact expression for scalar amplitude known:

$$A(\bar{\phi}_1,g_2^+,\ldots,\phi_n) = \frac{i2^{n/2-1}m^2\,\langle 2+|\Pi_{j=3}^{n-2}\,(y_{1,j}-\not{k}_j\not{k}_{1,j-1})\,|(n-1)-\rangle}{y_{1,2}\ldots y_{1,n-2}\,\langle 23\rangle\,\langle 34\rangle\ldots\langle (n-2)(n-1)\rangle}$$

($k_{1,j}=\sum_1^j k_j$, $y_{1,j}=k_{1,j}^2-m^2$) (Ferrario, Rodrigo, Talavera 06)

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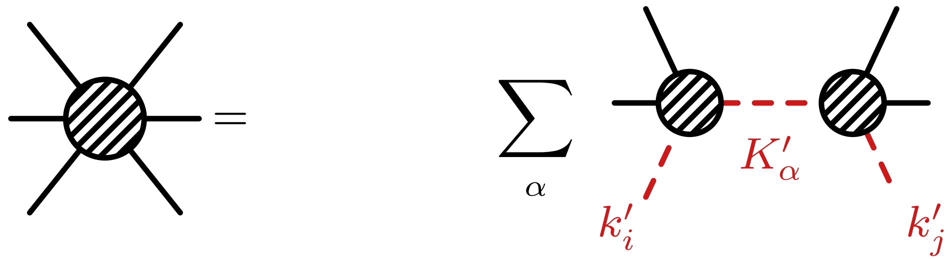
One negative helicity gluon:

Additional gluino contribution drops out for $|q+\rangle=|j+\rangle\ \Rightarrow$

$$A(\bar{Q}_1^+,\dots,g_j^-,\dots,Q_n^+)|_{|q+\rangle=|j+\rangle}=0$$

$$A(\bar{Q}_1^+,\dots,g_j^-,\dots,Q_n^-)|_{|q+\rangle=|j+\rangle}=\frac{\langle nj\rangle}{\langle 1j\rangle}A_n(\bar{\phi}_1^+,\dots,g_j^-,\dots\phi_n^-)$$

Construct amplitudes from on-shell sub-amplitudes



Shifted on-shell momenta: (Britto, Cachazo, Feng/ Witten, 04/05)

$$k'_i = k_i - z_\alpha \eta \qquad k'_j = k_i + z_\alpha \eta$$

with $\eta^2 = 0 \qquad k_i \cdot \eta = k_j \cdot \eta = 0$

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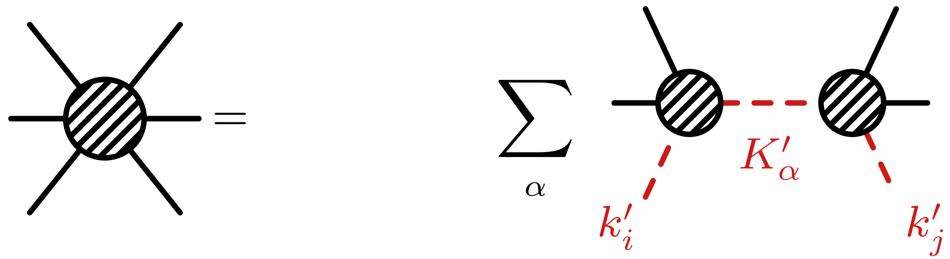

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$\Rightarrow \eta^\mu = \frac{1}{2} \langle i+|\gamma^\mu|j+\rangle$ for massless momenta

Corresponds to shifted spinors:

$$|i'+\rangle = |i+\rangle - z\,|j+\rangle \qquad |j'-\rangle = |j-\rangle + z\,|i-\rangle$$

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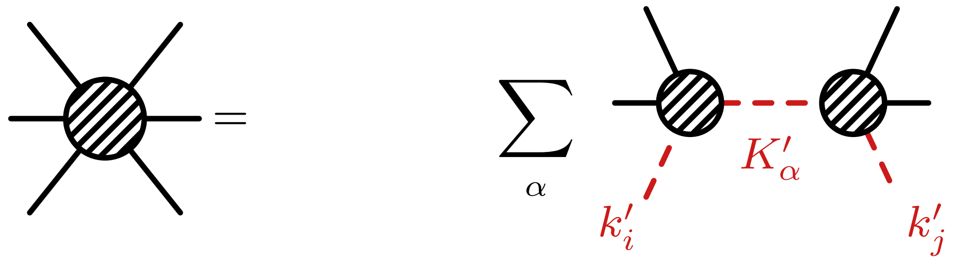

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Choose $z_\alpha = \dfrac{K_\alpha^2}{\langle i+|\not{K}_\alpha|j+\rangle} \quad \Rightarrow \quad K'^2_\alpha = 0$

Example: MHV amplitudes $A_n(g_1^+, \dots g_{(n-1)}^-, g_n^-)$

Use shift $|1'+\rangle = |1+\rangle - z\,|n+\rangle \qquad |n'-\rangle = |n-\rangle + z\,|1-\rangle$

- Amplitudes with only one g^- vanish
- Exception: 3-point amplitude: $0 = {k'_{1,2}}^2 = 2(k'_1 \cdot k_2) = \langle 1'2\rangle\,[21]$

$\Rightarrow$ Three point MHV vanishing, conjugate non-vanishing

One term contributing: (use $\langle n-|\,\not{k}'_{1,2} = \langle n-|\,\not{k}_{1,2}$ etc.)

$$A_n(g_1^+, \dots g_{(n-1)}^-, g_n^-) = A_3({g'}_1^+, g_2^+, g^-_{-k'_{1,2}})\frac{\mathrm{i}}{k_{1,2}^2} A_{n-1}(g^+_{k'_{1,2}}, \dots g_{(n-1)}^-, g_n^-)$$

$$= \mathrm{i}2^{n/2-1}\frac{[21]^3}{[k'_{1,2}2][1k'_{1,2}]}\frac{1}{\langle 12\rangle\,[21]}\frac{\langle (n-1)n\rangle^4}{\langle k'_{1,2}3\rangle\,\langle 34\rangle \dots \langle (n-1)n\rangle\,\langle nk'_{1,2}\rangle}$$

$$= \mathrm{i}2^{n/2-1}\frac{[12]^2}{\langle 12\rangle\,\langle n-|\not{k}_1|2+\rangle\,\langle 1+|\not{k}_2|3+\rangle}\frac{\langle (n-1)n\rangle^4}{\langle 34\rangle \dots \langle (n-1)n\rangle}$$

$$= \mathrm{i}2^{n/2-1}\frac{\langle (n-1)n\rangle^4}{\langle 12\rangle\,\langle 23\rangle \dots \langle n1\rangle}$$

Proof using complex continuation of scattering amplitude

$$A_n(z) = A_n(1', 2, \dots, (n-1), n')$$

On tree level: simple poles at $z_\alpha = -K_\alpha^2 / 2(\eta \cdot K_\alpha)$

$$\Rightarrow A_n(z) = \sum_{\text{poles } z_\alpha} \frac{c_\alpha}{z - z_\alpha} \quad \text{if } \lim_{z \to \infty} A_n(z) = 0$$

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Multiparticle poles of scattering amplitudes:

$$\lim_{z\to z_\alpha} A_n(z) = \sum_\lambda A(1', \ldots K'^{\lambda}_\alpha) \frac{\mathrm{i}}{K_\alpha^2 + 2z K_\alpha \cdot \eta} A(-K'^{-\lambda}_\alpha, \ldots n')$$

$$= -\frac{z_\alpha}{z - z_\alpha} \sum_\lambda A(1', \ldots K'^{\lambda}_\alpha) \frac{\mathrm{i}}{K_\alpha^2} A(-K'^{-\lambda}_\alpha, \ldots n') = \frac{c_\alpha}{z - z_\alpha}$$

$\Rightarrow$ BCWF relation:

$$A_n(0) = -\sum_{\alpha,\lambda} \frac{c_\alpha}{z_\alpha} = \sum_{\alpha,\lambda} A(1', \ldots K'^{\lambda}_\alpha) \frac{\mathrm{i}}{K_\alpha^2} A(-K'^{-\lambda}_\alpha, \ldots n')$$

Conditions for BCFW recursion:

$$A(z) \text{ has simple poles, } A(z) \to 0 \text{ for } z \to \infty$$

Most dangerous diagrams: only triple gluon vertices

$$A(z) \sim \underbrace{n\,\text{propagators}}_{z^{-n}} \times \underbrace{(n+1)\,\text{vertices}}_{z^{n+1}} \times \epsilon_i \times \epsilon_j \sim z \times \epsilon_i \times \epsilon_j$$

Consider shift $|i+\rangle' = |i+\rangle - z\,|j+\rangle \quad , \quad |j-\rangle' = |j-\rangle + z\,|i-\rangle$:

$$\epsilon^{\pm}_{\mu}(k', q) = \pm \frac{\langle q \mp |\gamma_\mu| k' \mp\rangle}{\sqrt{2}\,\langle q \mp | k' \pm\rangle} \sim \begin{cases} \epsilon^+(k_i') \sim z^{-1}, & \epsilon^-(k_i') \sim z \\ \epsilon^+(k_j') \sim z, & \epsilon^-(k_j') \sim z^{-1} \end{cases}$$

Sketch of proof of $A(z) \to 0$

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- (i^+, j^-): $A(z) \sim \frac{1}{z}$ from powercounting (BCFW 05)
  diagrammatic proof (Draggiotis et.al.; Vaman, Yao; 05)
- $(i^\pm, j^\pm)$: three particle auxiliary shift (Badger,Glover,Khoze,Svrček 05)
  follows from CSW representation (BCFW 05)

Color ordered amplitudes (Dindsdale, Ternik, Weinzierl, 06)

n	5	6	7	8	9	10
BG	0.23 ms	0.9	3	11	30	90
CSW*	0.4	4.2	33	240	1770	13000
BCFW	0.07	0.3	1	6	37	190

Averaged color ordered/dressed amplitudes (Duhr, Höche, Maltoni, 06)

n	5	6	7	8	9	10
BG (CD)	0.27 ms	0.72	2.37	8.21	27	86.4
BG (CO)	0.38	1.42	5.9	27.6	145	796
CSW (CO)†	0.6	2.78	14.6	91.9	631	4890
BCFW(CO)	0.26	1.2	7.4	59.7	590	6400

(*: recursive reformulation(Bena, Bern, Kosower, 04); †: reformulation with cubic vertices)

BCFW for one-loop amplitudes?

conditions for proof violated: (Bern, Dixon, Kosower 05)

- cuts $\Rightarrow$ not just single poles

$\Rightarrow$ look at rational part R only

- in general $\lim_{z\to\infty} A(z) \neq 0$
- for complex kinematics double poles $\sim [ij]/\langle ij\rangle^2$,
  "unreal" poles $\sim [ij]/\langle ij\rangle$
- Cannot avoid both contributions from $A(\infty)$ or double poles

General recipe (Berger, Bern, Forde, Dixon, Kosower 06)

- primary shift without double poles
- auxiliary shift to determine $z \to \infty$ contribution

BCFW recursion for massive scalars (Badger,Glover,Khoze,Svrček 05)

- applied for shifted gluon lines
- shifted massive momenta defined ... not yet applied

Massive fermions (+gauge bosons)

- "stripped" amplitudes: (Badger, Glover, Khoze 05)
  remove spinors of internal quark lines:

$$\sum_{\sigma=\pm} A(\dots, Q^{\sigma}_{K'}) \frac{\mathrm{i}}{K^2 - m^2} A(\bar{Q}^{-\sigma}_{K'}, \dots) = A(\dots, Q^{\bullet}_{K'}) \frac{\mathrm{i}(\not{K}' + m)}{K^2 - m^2} A(\bar{Q}^{\bullet}_{K'}, \dots)$$

- 5-6 point $Q\bar{Q}$ amplitudes calculated (Ozeren,Stirling 06; Hall 07)

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BCFW relations for all born QCD amplitudes?

- allowed helicities?
- shift of massive quark lines?

Decompose general momenta into light-like $l_{i/j}$: (del Aguila, Pittau 04)

$$p_i = l_i + \alpha_j l_j \quad , \quad p_j = \alpha_i l_i + l_j$$

with $$\alpha_\ell = \frac{2p_ip_j \mp \sqrt{\Delta}}{2p_\ell^2}, \qquad \Delta = (2p_ip_j)^2 - 4p_i^2p_j^2$$

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Define shifted spinors: (CS, S.Weinzierl 07)

$$u_i{}'(-) = u_i(-) - z\,|l_j+\rangle \quad , \quad \bar{u}_j'(+) = \bar{u}_j(+) + z\,\langle l_i+|$$

with reference spinors $|q_i\pm\rangle = |l_j\pm\rangle$, $|q_j\pm\rangle = |l_i\pm\rangle$

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Corresponds to shifted momenta

$$p_i'^{\mu} = p_i^\mu - \frac{z}{2}\langle l_i+|\gamma^\mu|l_j+\rangle \ , \quad p_j'^{\mu} = p_j^\mu + \frac{z}{2}\langle l_i+|\gamma^\mu|l_j+\rangle$$

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Remark: Without fixing q one gets spurious poles in z:

$$u_i'(-) \stackrel{?}{=} \frac{(\not{p}_i' + m)\,|q-\rangle}{[p_i^\flat q]} \qquad \bar{u}_i'(+) \stackrel{?}{=} \frac{\langle q-|\,(\not{p}_i' + m)}{\langle q p_i^\flat\rangle - z\,\langle q l_j\rangle}$$

Recursion relation:

$$A_n(1,\dots,i,\dots,j,\dots,n) =$$

$$\sum_{\text{partitions},h=\pm} A_L(\dots,i',\dots K'^{h},\dots)\frac{i}{K^2-m_k^2}A_R(\dots,-K'^{-h},\dots j',\dots)$$

Intermediate massive quark: choose $\langle q_K-| = \langle l_j-|$ and $|q_K-\rangle = |l_i-\rangle$:

$$u'_K(-) = \frac{1}{\langle K^\flat + |l_i-\rangle}(\not{K}+m_k)\,|l_i-\rangle \qquad \bar{u}'_K(+) = \frac{1}{\langle l_j - |K^\flat+\rangle}\langle l_j-|\,(\not{K}+m_k)$$

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Conditions for $\lim_{z\to\infty} A(z) \to 0$

- (i^+, j^-) allowed if Q_i and Q_j are not joined by quark line

  (as for massless quarks: Luo, Wen; Badger et.al; Quigly, Rozali; 05)

- (g_i^+, g_j^+), (g_i^+, Q_j^+) , (g_i^-, g_j^-), (Q_i^-, g_j^-) allowed

- for (Q_i^+, Q_j^+), (Q_i^-, Q_j^-) three particle shift necessary

Application: Amplitudes with g_2^- from shift $(i,j) = (Q_1^{\pm}, g_2^-)$:

$$\bar{u}_1'(-) = \bar{u}_1(-) - z\,\langle 2-| \qquad , |2'-\rangle = |2-\rangle + z\,|l_1-\rangle$$

Amplitude expressed in terms of known quantities:

$$A_n(\bar{Q}_1^{\lambda_1}, g_2^-, g_3^+ \ldots, Q_n^{\lambda_n}) =$$

$$\sum_{j=3}^{n} A(\bar{Q}'^{\lambda_1}_1, g^+_{k'_{2,j}}, g^+_{j+1}, \ldots, Q_n^{\lambda_n}) \frac{i}{k_{2,j}^2} A_{\text{MHV}}(g^-_{-k'_{2,j}}, g_2'^{\,-}, \ldots, g_j^+)$$

Application: Amplitudes with g_2^- from shift $(i,j) = (Q_1^\pm, g_2^-)$:

$$\bar{u}'_1(-) = \bar{u}_1(-) - z\,\langle 2-| \qquad , |2'-\rangle = |2-\rangle + z\,|l_1-\rangle$$

Amplitude expressed in terms of known quantities:

$$A_n(\bar{Q}_1^{\lambda_1}, g_2^-, g_3^+ \dots, Q_n^{\lambda_n}) =$$

$$\sum_{j=3}^{n} A(\bar{Q}'^{\lambda_1}_1, g^+_{k'_{2,j}}, g^+_{j+1}, \dots, Q_n^{\lambda_n}) \frac{i}{k_{2,j}^2} A_{\text{MHV}}(g^-_{-k'_{2,j}}, g'^-_2, \dots, g_j^+)$$

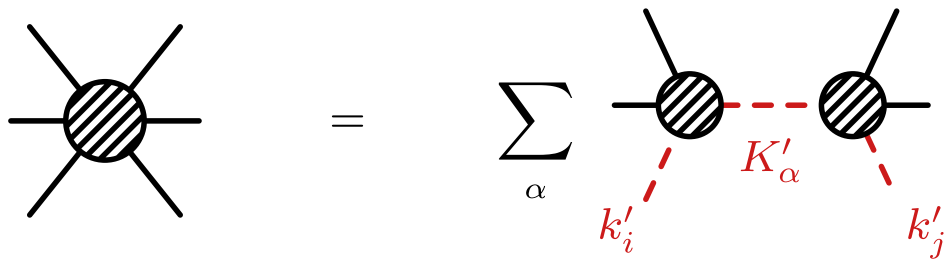
Example: with $|\Phi_{k,n}-\rangle = \prod_{j=k}^{n-2}\left(1 - \frac{\not{p}_j \not{p}_{1,j}}{y_{1,j}}\right)|(n-1)-\rangle\,.$

$$A_n(\bar{Q}_1^+, g_2^-, \dots, Q_n^-) = \frac{i2^{n/2-1}\langle n^\flat 2\rangle}{\langle 1^\flat 2\rangle\langle 23\rangle \dots \langle (n-2)(n-1)\rangle}$$

$$\sum_{j=3}^{n-1} \frac{\langle 2-|\not{k}_1\not{k}_{2,j}|2+\rangle^2}{k_{2,j}^2\,\langle 2-|\not{k}_1\not{k}_{2,j}|j+\rangle}\left(\delta_{j,n-1} + \delta_{j\neq n-1}\frac{m^2\,\langle 2-|\not{k}_{2,j}|\Phi_{j+1,n}-\rangle\,\langle j(j+1)\rangle}{y_{1,j}\,\langle 2-|\not{k}_1\not{k}_{2,j}|(j+1)+\rangle}\right)$$

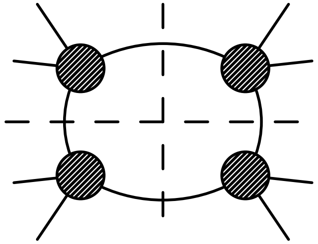
Simpler calculation than from shift of gluons (Forde, Kosower 05)

BCFW-Summary



- Recursive construction of scattering amplitudes from on-shell sub-amplitudes
- Loops: rational part of amplitudes
- Extension to massive quarks
  - Shift of massive quark lines
  - Clarified allowed helicities

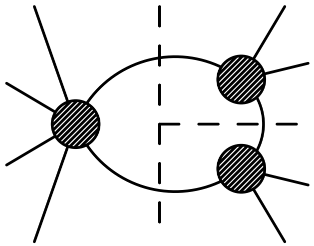
Box coefficients from quadruple cuts (Britto, Cachazo, Feng 04)



$$\Rightarrow c_{ijk}^{(4)} \sim \sum_{k_{1/2}} A_{1,i}\, A_{i,j}\, A_{j,k}\, A_{k,n}$$

complex momenta to solve constraints

Triangle/Box coefficients from triple/double cuts (Forde 07)



$$\Rightarrow c_{ij}^{(3)} = -\mathsf{Inf}[A_{1,i}\, A_{i,j}\, A_{j,n}]_{t=0}$$

$$\lim_{t\to\infty} (\mathsf{Inf}[f(t)] - f(t)) = 0$$

parameterization of loop-momentum $l = a_0 + t a_1 + t^{-1} a_2$

General masses Tadpoles (Kilgore 07)

Numerical methods (Ossola, Papadopoulos, Pittau 06; Ellis, Giele, Kunszt 07)

Helicity methods in QCD: color ordering , MHV amplitudes, Berends-Giele recursion

New methods: CSW diagrams, on-shell recursion relations, new unitarity methods

Extension to massive particles

- new CSW vertex for massive scalars
- SUSY-relations of massive quarks to massive scalars
- On-shell recursion for massive quarks

Impact on phenomenology?

- numerical tree-level calculations: Berends-Giele wins asymptotically, but BCFW competitive for $n \leq 7-8$
- unitarity methods suitable for automatization, numerical stability remains to be checked

Stay tuned for more surprises