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How the Quantum Doctor Treats the Collapse
(Of the Wave Function)

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Following H. Nikolic's talk in the Colloquium of April 12, 2007 on the Bohmian interpretation of QM we had a lot of (informal) discussions in the group – usually at coffee time ...

Here is a small contribution to a better understanding of the measuring process ...

It is an amateur's understanding ... based on the paper

"The quantum measurement process: an exactly solvable model"

by

A. Allahverdyan, R. Balian & Th. Nieuwenhuizen

[arXiv: cond-mat/0309188]

Additional reading:

"On the interpretation of quantum theory
– from Copenhagen to the present day"

by

C. Kiefer

[arXiv: quant-ph/0210152]

Wikipedia articles on:

"Wave function collapse"

"Copenhagen interpretation"

"Quantum decoherence"

[has the warning: "This article or section may be **confusing or unclear** for some readers"]

⋮

Outline:

1. Wave function collapse in the Copenhagen interpretation
2. The model
3. Coupling to the environment
4. Results
5. Summary

1. The Copenhagen interpretation

"There is no definitive statement of the Copenhagen interpretation ...

Principles

1. A system is completely described by a wave function ψ , which represents an observer's knowledge of the system (Heisenberg)

2. The description of nature is essentially probabilistic. The probability of an event is related to the square of the amplitude of the wave function (Born)

(3.) Heisenberg's **uncertainty principle** ensures that it is not possible to know the values of all of the properties of the system at the same time ...

(4.) **Complementary Principle**: matter exhibits a wave-particle duality ... (Bohr)

5. Measuring devices are essentially **classical** devices, and measure classical properties such as position and momentum

(6.) **Correspondence Principle**: the quantum mechanical description of *systems with large quantum numbers should approach* [my version] the classical description. (Bohr & Heisenberg)

Niels Bohr emphasized that Science is concerned with the predictions of experiments, additional questions are not scientific but rather meta-physical ... "

Measurement process and wave function collapse

According to Copenhagen principle 5 the world is divided into a quantum world (system) and a classical world (apparatus)

In the quantum world the time evolution is deterministic, unitary and continous:

$$|\psi(t)> = \, e^{-i\hat{H}(t-t_0)/\hbar} \, |\psi(t_0)>$$

von Neumann ("Mathematische Grundlagen der Quantenmechanik", 1932) introduced as further postulate that an (ideal) measurement at time t_1 associated with the hermitean operator $\hat{A}$ reduces the wavefunction to just one component

$$|\psi(t_1)> \quad = \quad \sum_n |n> \underbrace{<n|\psi(t_1)>}_{\equiv c_n} ; \quad \hat{A}|n> = A_n|n>$$

$$\xrightarrow{\text{measurement}} \quad c_n|n>$$

and the particular value A_n of the observable $\hat{A}$ is measured with probablity

$$|c_n|^2 = |< n|\psi(t_1) >|^2$$

In other words:

There is a non-unitary , non-local, discontinous change of the system brought about by observation !

This is very unnatural and unsatisfactory ...

Quantum physics should also describe the measurement process, i.e. the apparatus

Interpretations of QM **without** wave function collapse:

- Pilot waves (de Broglie, Bohm)
- Many worlds (Everett)
- Consistent histories (Griffiths, Hartle, Gell-Mann)

2. The model

System = spin s_z of one particle

Apparatus = magnet (M) coupled to a bath (B)

Magnet contains N spins $\sigma_z^{(n)} = \pm 1$ with (mean-field) interaction

$$H_M = -\frac{J}{4N^3} \sum_{ijkl=1}^{N} \sigma_z^{(i)} \sigma_z^{(j)} \sigma_z^{(k)} \sigma_z^{(l)} \equiv -\frac{1}{4} N J m^4$$

where

$$m = \frac{1}{N} \sum_{n=1}^{N} \sigma_z^{(n)}$$

is the magnetization

Interaction between test system (S) and apparatus (A)

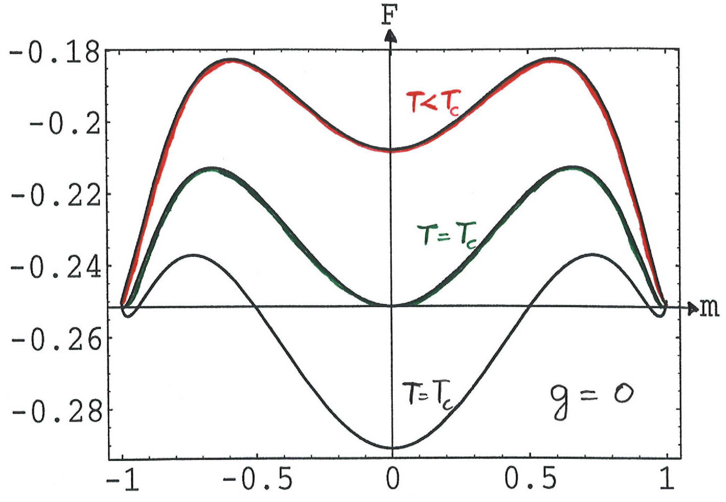
$$H_{SA} = -g s_z \sum_{n=1}^{N} \sigma_z^{(n)} = -g s_z N m$$

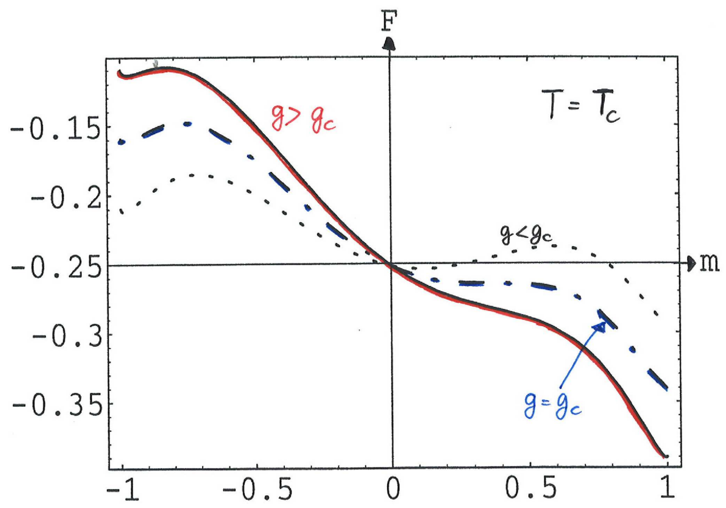
is turned on at time $t_1 = 0$ (begin of measurement) and turned off at time t_2 (end of measurement)

First classical treatment: spins are Ising spins taking vakues ± 1

Free energy per spin at given temperature T can be calculated exactly (?)

$$F = -\frac{1}{4} J m^4 - g s_z m - T \left(\frac{1+m}{2} \ln \frac{2}{1+m} + \frac{1-m}{2} \ln \frac{2}{1-m} \right)$$


F
-0.18
-0.2
-0.22
-0.24
-0.26
-0.28
-1
-0.5
0
0.5
1
m
T<Tc
T=Tc
T=Tc
g = 0


F
-0.15
-0.2
-0.25
-0.3
-0.35
-1
-0.5
0
0.5
1
m
T = Tc
g> gc
g<gc
g = gc

Discussion:

- At large temperature the magnet is in the paramagnetic state: spins are randomly up or down $\Longrightarrow$ average magnetization $m = 0$
- At critical temperature $T_c = 0.36295\,J$ the magnet undergoes a (first-order) transition to a state with magnetization $\pm m_c$
- At $g = 0$ and $T < T_c$ the paramagnetic state $m = 0$ is still **metastable**

In this metastable state the apparatus is prepared for the measurement of the spin of the particle:

magnetic analog of the oversaturated gas in a cloud or bubble chamber

Measurement

At time $t_1 = 0$ the coupling between test-spin and the magnet is turned on:

this is equivalent to putting the magnet in an external field $gs_z = \pm g$

If g is large enough and $s_z = +1$, the interaction suppresses the barrier near $m \approx 0.7$ and for $s_z = -1$ it will suppress the one near $m \approx -0.7$

$\Longrightarrow$ the magnetization will move from $m = 0$ to the minimum of the free energy F

... and will stay there provided the available energy is dissipated,

i.e. given to the environment

How does one describe dissipation (friction) in QM ?

3. Coupling to the environment

Simple damping as for a classical harmonic oscillator

$$\ddot{x} + \omega^2 x = 0 \longrightarrow \ddot{x} - \gamma \dot{x} + \omega^2 x = 0$$

cannot be used since it requires a non-unitary Hamiltonian !

Main idea (Feynman & Vernon, Caldeira & Leggett): consider

system + environment (modelled as a "bath" of M harmonic oscillators)

+ bi-linear coupling between the two parts

$$H = \frac{p^2}{2m} + \frac{1}{2} m\, \omega^2\, q^2 + \sum_{n=1}^{M} \left(\frac{p_n^2}{2m_n} + \frac{1}{2} m_n\, \omega_n^2 x_n^2 \right) - q \sum_{n=1}^{M} b_n\, x_n$$

and integrate out (exactly !) the bath's degree's of freedom

Obtain effective two-time action (memory effect !) for the particle with damping kernel

$$\gamma(t) = \frac{1}{m}\sum_{n=1}^{M}\frac{b_n^2}{m_n\omega_n^2}\cos(\omega_n t) \quad \stackrel{M\to\infty}{\longrightarrow} \quad \frac{2}{m\pi}\int_0^\infty d\omega\,\frac{J(\omega)}{\omega}\cos(\omega t)$$

where in the limit $M \to \infty$ $J(\omega)$ is the continous spectral density of the environment oscillators.

A simple parametrization

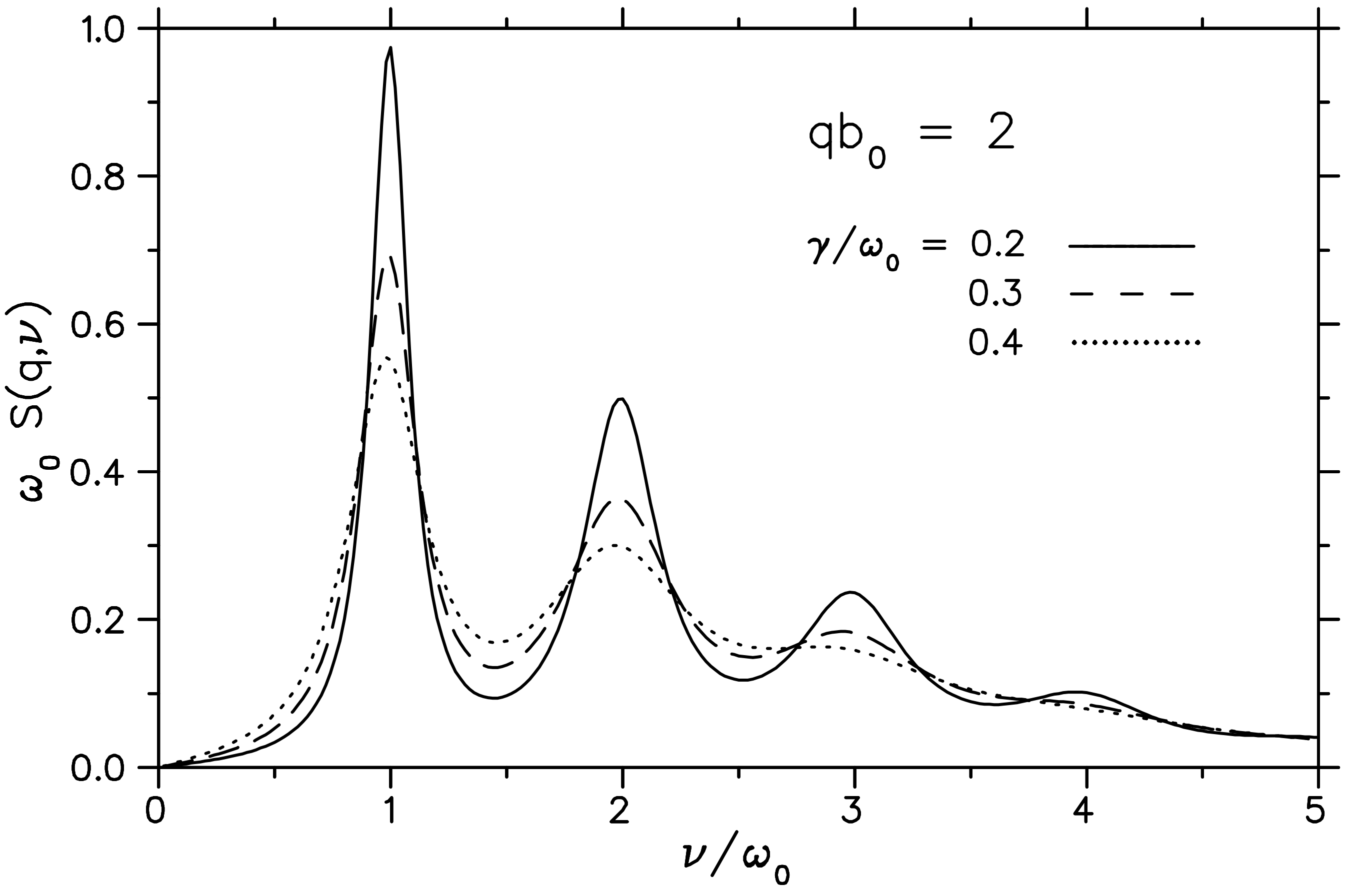
$$J^{\rm Ohm}(\omega) = m\,\gamma\,\omega \implies \gamma(t) = 2\delta(t)\,\gamma$$

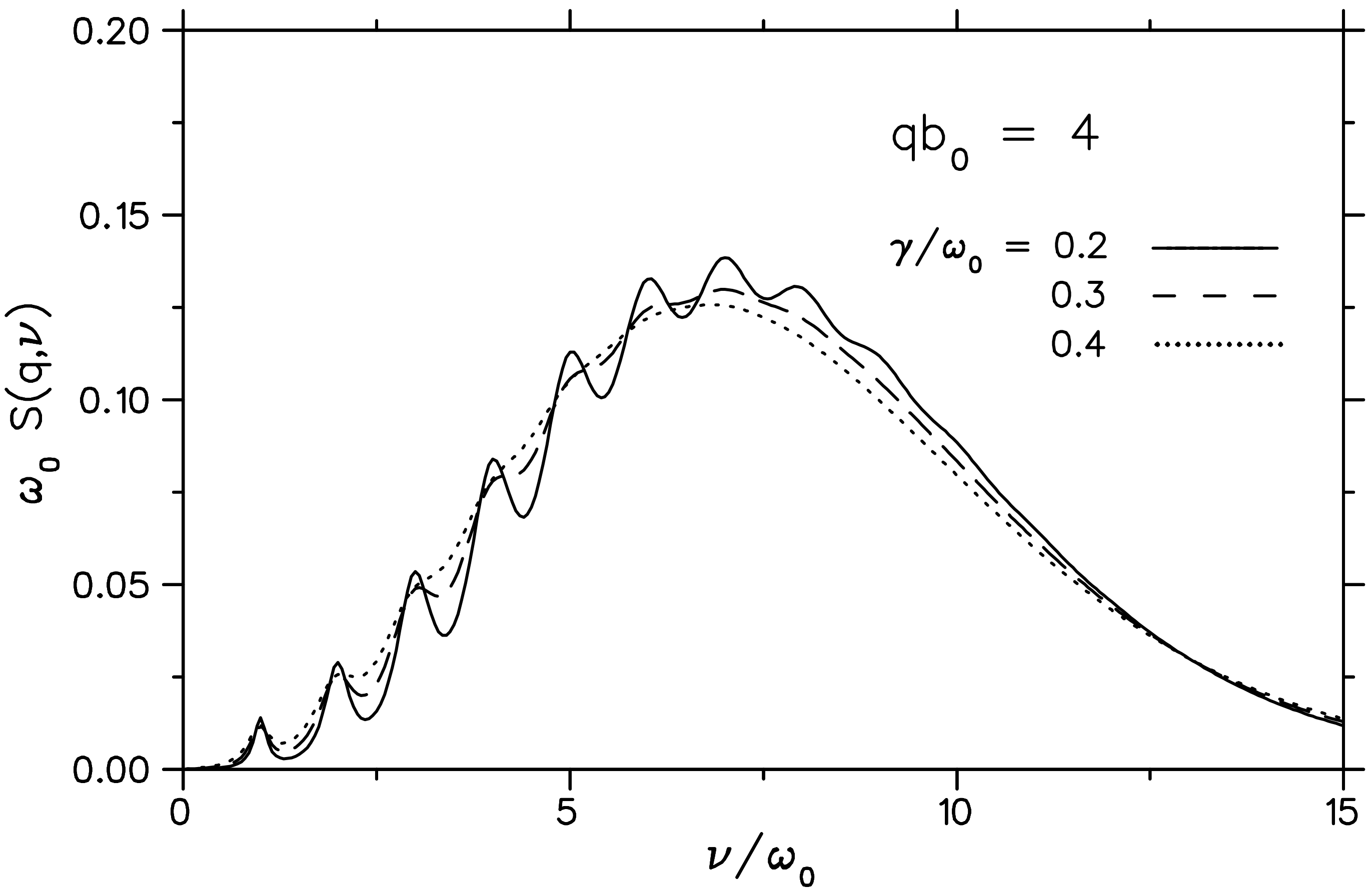
gives classical damping without memory

Applications:

1. A model for inclusive scattering from harmonically confined quarks (RR, Phys. Rev. **C 68** (2003)): consistent hermitean description $\Longrightarrow$ no violation of sum rules despite loss of flux into unobserved channels

2. In the present model: all 3 spin components of all N apparatus spins are weakly coupled to an Ohmic bath


$qb_0 = 2$
$\gamma/\omega_0 = 0.2$
0.3
0.4
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
$\omega_0\ S(q,\nu)$
ν/ω_0



qb₀ = 4
γ/ω₀ = 0.2
0.3
0.4
ω₀ S(q,ν)
ν/ω₀
0.00
0.05
0.10
0.15
0.20
0
5
10
15

4. Results

If the coupling g between the test-spin and the magnet is large enough

$$g > g_c = 0.09035\, J$$

then the free energy barrier can be overcome and

- for $s_z = +1$ the magnet will end up in the right (true) minimum;
- for $s_z = -1$ the magnet ends up in the left (true) minimum

After m has approached the minimum, the apparatus is decoupled ($g \to 0$). i.e. the measurement is finished

The magnetization will then move to the $g = 0$-minimum which is about 0.004 deeper

It will stay there up to a hopping (Poincaré) time $\propto e^M$; for large M this means **"forever"** .

Whether or when the apparatus is read off ("observation") is irrelevant !

The collapse (quantum mechanical treatment)

The test spin may start in an unknown quantum state:

$$< s_x > , < s_y > , < s_z > \qquad \text{are arbitrary}$$

$\Longrightarrow$ initial **density matrix** (reminder: for a pure state $r = |\psi >< \psi| \longrightarrow r^2 = r$)

$$r(0) = \frac{1}{2} \begin{pmatrix} 1+ < s_z > & < s_x > -i < s_y > \\ < s_x > +i < s_y > & 1- < s_z > \end{pmatrix}$$

Full initial density matrix of system

$$\mathcal{D}(0) = r(0) \otimes R_M(0) \otimes R_B(0)$$

where

$$R_M(0) = \prod_{n=1}^{N} \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

is the density matrix of the random spins of the magnet in the paramagnetic (metastable) state ($m = 0$)

Time evolution

$$i\hbar \frac{d}{dt}\mathcal{D}(t) = [H, \mathcal{D}]$$

where H is the full Hamiltonian of system + apparatus + environment.

The state of the system (= test spin) is given by the reduced density matrix

$$r(t) = \text{tr}_{M,B}\mathcal{D}(t)$$

and its time evolution comes from the interaction Hamiltonian H_{SA}

$$\frac{d}{dt}r_{ij}(t) = -gN\,(s_i - s_j)\,\text{tr}_{M,B}\,[m, \mathcal{D}_{ij}(t)]$$

where $i, j = \uparrow, \downarrow$, $s_\uparrow = +1$, $s_\downarrow = -1$

Note: diagonal elements are conserved in time:

$$r_{\uparrow\uparrow}(t) = p_\uparrow = \frac{1}{2}(1 + < s_z >) = \text{const.}$$
$$r_{\downarrow\downarrow}(t) = p_\downarrow = \frac{1}{2}(1 - < s_z >) = \text{const.}$$

only off-diagonal elements are endangered and actually will collapse !

Solution :

For short times the spin-spin interactions in the magnet and the spin-bath interactions are still neglible (?) $\longrightarrow$ problem reduces to the evolution of N independent apparatus spins coupled to the test spin

Thus for each individual density matrix in the magnet one obtains

$$\frac{1}{2}\begin{pmatrix} \exp(2igt/\hbar) & 0 \\ 0 & \exp(-2igt/\hbar) \end{pmatrix}$$

Then

$$\begin{aligned} r_{\uparrow\downarrow}(t) &= r_{\uparrow\downarrow}(0)\left[\cos\frac{2gt}{\hbar}\right]^N \approx r_{\uparrow\downarrow}(0)\left[1-2\left(\frac{gt}{\hbar}\right)^2+\dots\right]^N \\ &\approx r_{\uparrow\downarrow}(0)\exp\left(-2\frac{g^2N}{\hbar^2}t^2\right) \end{aligned}$$

Therefore the off-diagonal parts of the density matrix of the test spin will collapse within a time

$$\tau_{\text{collapse}} = \frac{\hbar}{g\sqrt{2N}} = \frac{\text{const}}{\sqrt{N}}$$

If we estimate $g \sim J \sim T$ and assume $N \gg 1$ then

$$\tau_{\text{collapse}} \ll \frac{\hbar}{T} \equiv \tau_{\text{thermal fluctuation}}$$

Note: the recurrent peaks of the cosine at $t_k = k\pi\hbar/(2g)$, $k = 1, 2 \ldots$ are

suppressed by the coupling to the bath which brings a factor (?)

$$e^{-\gamma\hbar N} \stackrel{N\gg 1}{\longrightarrow} 0$$

Registration of the quantum measurement

After the off-diagonal sectors of the density matrix have decayed the diagonal ones evolve under the influence of H_{SA} **and** the coupling between the apparatus and the environment.

One finds (?) that the net magnetization of the magnet obeys the following equation of motion

$$\frac{d}{dt}\langle m\rangle \;=\; \gamma\, h \left(1 - \frac{< m >}{\tanh(h/T)} \right) \;,\; h \;:=\; g < m > s_z + J < m >^3$$

exactly (?) as in a classical treatment of the transition from the metastable paramagnetic state to the true (ferromagnetic) groundstate

characteristic time scale for this transition($\gamma \ll 1/\hbar$)

$$\tau_{\text{registration}} \;=\; \frac{1}{\gamma g} \sim \frac{1}{\gamma J} \gg \frac{\hbar}{T} \equiv \tau_{\text{thermal fluctuation}}$$

Result of measurement

After the measurement, at $t_2 \gg \tau_{\text{registration}}$, the common state of test spin and apparatus is

$$D(t_2) = p_\uparrow \, |\uparrow\rangle \langle\uparrow| \otimes \rho^{(1)}_{\uparrow\uparrow}(t_2) \otimes \ldots \otimes \rho^{(N)}_{\uparrow\uparrow}(t_2) + p_\downarrow \, |\downarrow\rangle \langle\downarrow| \otimes \rho^{(1)}_{\downarrow\downarrow}(t_2) \otimes \ldots \otimes \rho^{(N)}_{\downarrow\downarrow}(t_2)$$

where

$$p_\uparrow = \frac{1}{2}(1+ < s_z >) \; , \; p_\downarrow = \frac{1}{2}(1- < s_z >)$$

$$\rho^{(n)}_{\uparrow\uparrow}(t_2) = \frac{1}{2} \begin{pmatrix} 1 + m_\uparrow & 0 \\ 0 & 1 - m_\uparrow \end{pmatrix}$$

In words: with probability $p_\uparrow$ one finds the spin in the up-state and the magnet with magnetization up and like-wise in the down-sector The off-diagonal sectors (“Schrödinger cats”) have been eliminated by the collapse: decoherence

Statistical interpretation: (still needed ?)

Assume:

Any quantum state (density matrix) describes an **ensemble of states** **not** an individual particle. Quantum measurement = a series of measurements on an ensemble of identically prepared sytems

Quote:

"In doing a series of experiments, there are two possible outcomes, connected with the magnetization of the apparatus being up or down, which occur with probabilities $p_\uparrow$ and $p_\downarrow$, respectively. In each such event, the z-component of the test spin is equal to $+1$ or -1, correspondingly. The ensemble of spins having $+1$ is described by the **pure** density matrix $|\uparrow><\uparrow|$ or simply by the wavefunction $|\uparrow>$. A similar statement holds for the down spins."

5. Summary

- Allaverdhyan *et al.* have given a simplified, semi-realistic **model of the quantum measurement process** (for a test sytem) in which the apparatus is treated quantum mechanically as well and coupled to an environment (bath). The number N of spins in the apparatus is arbitrary which allows to study the macroscopic limit

- Von Neumann was right (this time): the **collapse occurs quite fast** after the start of the measurement **if N is large**

- **It goes in 2 steps**: the collapse occurs due to the interaction of the test system with the macroscopic apparatus and later is made definite by bath induced **decoherence**

- The **registration of the measurement** occurs in a "classical" state, i.e. a state that has collapsed already

- Whether the outcome is **observed or not** is immaterial. Gravitation plays no role. Extensions of QM (like non-linear Schrödinger eqs. for a spontaneous collapse) are not needed

- Approach can (in principle) be tested by observing N-dependence of collapse time in mesoscopic devices (?)