Three - and Four - Parton Contributions to the Heavy Quark Forward - Backward - Asymmetry

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Why Forward - Backward - Asymmetry?

- high precision analysis of the electroweak part of the Standard Model
- A_{FB} for massive quark production one of the most precise determinations of $\sin\theta_W$
- high precision of experimental data

$\longrightarrow$ theoretical predictions for higher-order electroweak and QCD corrections are needed

Process

Forward-Backward Asymmetry of the process $e^- + e^+ \xrightarrow{\gamma,Z} t + \overline{t} + X$

Possibilites for X up to the order $\mathcal{O}(\alpha_S^2)$

Three - Parton Finalstate:

$X = g$

Four - Parton Finalstate:

$X = q + \overline{q}$ with massless quarks q

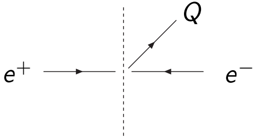
$X = g + g$

$X = t + \overline{t}$

A_{FB} Part I

Definition of A_{FB}

A_{FB} is defined as the number of quarks Q observed in the forward hemisphere minus the number of quarks Q in the backward hemisphere, divided by the total number of observed quarks Q



$$\sigma_F = \int_0^1 \frac{d\sigma}{dcos\theta} dcos\theta, \quad \sigma_B = \int_{-1}^0 \frac{d\sigma}{dcos\theta} dcos\theta \tag{1}$$

A_{FB} Part II

Using σ_F and σ_B, A_{FB} is given by

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{\sigma_A}{\sigma_S} \quad (2)$$

with $\sigma_A = \sigma_F - \sigma_B$ and $\sigma_S = \sigma_F + \sigma_B$

Advantage: IR safety of A_{FB}

Common choices for the axis

- direction of flight of Q
- thrust axis direction: $\vec{T} = T\vec{n_T}$

$$T = \max_{\vec{n}} \frac{\sum_{i=1}^{m} \|\vec{p}_i \vec{n}\|}{\sum_{i=1}^{m} \|\vec{p}_i\|} \quad (3)$$

where $\vec{p_i}$ are the momenta of the final state partons.

Order α_S^2 contributions to A_{FB}

Consider the process $e^- e^+ \xrightarrow{\gamma, Z} Q\overline{Q} + X$
To order α_S^2 the symmetric and antisymmetric cross section receive the following contributions:

$$\sigma_{A,S} = \sigma_{A,S}^{2,0} + \sigma_{A,S}^{2,1} + \sigma_{A,S}^{2,2} + \sigma_{A,S}^{3,1} + \sigma_{A,S}^{3,2} + \sigma_{A,S}^{4,2} + \mathcal{O}(\alpha_S^3) \qquad (4)$$

where the first number in the superscripts (i, j) denotes the number of final state partons and the second one the order of α_S

$$A_{FB}(\alpha_S^2) = \frac{\sigma_A^{2,0} + \sigma_A^{2,1} + \sigma_A^{2,2} + \sigma_A^{3,1} + \sigma_A^{3,2} + \sigma_A^{4,2}}{\sigma_S^{2,0} + \sigma_S^{2,1} + \sigma_S^{2,2} + \sigma_S^{3,1} + \sigma_S^{3,2} + \sigma_S^{4,2}} \qquad (5)$$

Convenient to rewrite Eq. (5) as:

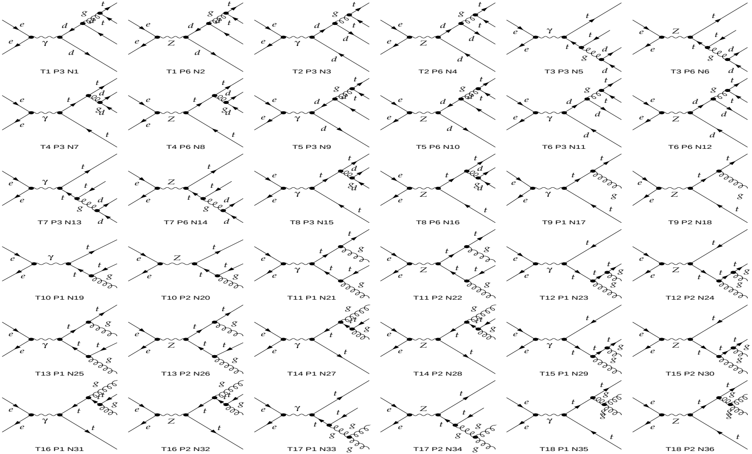
$$A_{FB}(\alpha_S^2) = A_{FB}^{(2p)} + A_{FB}^{(3p)} + A_{FB}^{(4p)} \qquad (6)$$

Order α_S^2 contributions to A_{FB}

$$A_{FB}(\alpha_S^2) = A_{FB}^{(2p)} + A_{FB}^{(3p)} + A_{FB}^{(4p)} \quad (7)$$

- $A_{FB,(3p)}^{(3,1)}$ is finite
- Sum of $A_{FB,(3p)}^{3,2}$ and $A_{FB,(4p)}^{4,2}$ is finite
  - no double unresolved particles
  - Dipole Subtraction terms only needed for $A_{FB}^{(4p)}$

Diagrams


T1 P3 N1
T1 P6 N2
T2 P3 N3
T2 P6 N4
T3 P3 N5
T3 P6 N6
T4 P3 N7
T4 P6 N8
T5 P3 N9
T5 P6 N10
T6 P3 N11
T6 P6 N12
T7 P3 N13
T7 P6 N14
T8 P3 N15
T8 P6 N16
T9 P1 N17
T9 P2 N18
T10 P1 N19
T10 P2 N20
T11 P1 N21
T11 P2 N22
T12 P1 N23
T12 P2 N24
T13 P1 N25
T13 P2 N26
T14 P1 N27
T14 P2 N28
T15 P1 N29
T15 P2 N30
T16 P1 N31
T16 P2 N32
T17 P1 N33
T17 P2 N34
T18 P1 N35
T18 P2 N36

Spinor Helicity Formalism

Massless Fermions

Define

- $|k^{\pm}\rangle \equiv u_{\pm}(k) = \frac{1}{2}(1 \pm \gamma_5)u(k)$
- $\langle k^{\pm}| \equiv \overline{u_{\pm}(k)} = \overline{u(k)}\frac{1}{2}(1 \mp \gamma_5)$

Basic Spinor products

- $\langle ij\rangle = \langle i^{-}|j^{+}\rangle = \overline{u_{-}(k_i)}u_{+}(k_j)$
- $[ij] = \langle i^{+}|j^{-}\rangle = \overline{u_{+}(k_i)}u_{-}(k_j)$
- all other products vanish because of helicity projection

Spinor Product

For numerical evaluation of the spinor products obtain the explicit formula

$$\langle ij \rangle = \sqrt{k_{i-} k_{j+}} e^{i\phi_{k_i}} - \sqrt{k_{i+} k_{j-}} e^{i\phi_{k_j}} \tag{8}$$

$$[ij] = \sqrt{k_{i+} k_{j-}} e^{-i\phi_{k_j}} - \sqrt{k_{i-} k_{j+}} e^{-i\phi_{k_i}} \tag{9}$$

with

$$e^{\pm i\phi_k} = \frac{k^1 \pm i k^2}{\sqrt{k_+ k_-}}, \quad k_\pm = k^0 \pm k^3 \tag{10}$$

Properties of the Spinor Product

- Antisymmetry $\langle ij \rangle = -\langle ji \rangle, \quad [ij] = -[ji]$
- Charge Conjugation $\langle i^+ | \gamma^\mu | j^+ \rangle = \langle j^- | \gamma^\mu | i^- \rangle$
- Fierz Identity $\langle i^+ | \gamma_\mu | j^+ \rangle \langle k^- | \gamma^\mu | l^- \rangle = 2 \langle il \rangle [kj]$

Spinor Product

Introduce a spinor representation of the polarization vector of a massless gauge boson

$$\epsilon_\mu^{\pm}(k,q) = \frac{\langle q^{\mp}|\,\gamma_\mu\,|k^{\mp}\rangle}{\sqrt{2}\,\langle q^{\mp}|\,|k^{\mp}\rangle} \tag{11}$$

with reference momentum q $\quad q \neq k$

Properties

- $\left(\epsilon_\mu^{+}\right)^* = \epsilon_\mu^{-}$
- $\epsilon^{+} \cdot \left(\epsilon^{+}\right)^* = -1$
- $\epsilon^{+} \cdot \left(\epsilon^{-}\right)^* = 0$
- $\sum_{\lambda=\pm} \epsilon_\mu^{\lambda}(k,q)\left(\epsilon_\nu^{\lambda}(k,q)\right)^* = -g_{\mu\nu} + \frac{k_\mu q_\nu + k_\nu q_\mu}{k \cdot q}$

Spinor Helicity Formalism

Spinor representation for massive fermions, such that

$$\sum_{s=1,2} u(k,s)\overline{u}(k,s) = \not{k} + m \tag{12}$$

is reproduced

decompose k^μ into two light-like momenta l_1^μ, l_2^μ

$$l_1^2 = 0 = l_2^2, \quad l_1^\mu + l_2^\mu = k^\mu. \tag{13}$$

- $u_\pm(p) = \frac{[l_1 l_2]}{m} |l_1^\pm\rangle + |l_2^\mp\rangle$
- $\overline{u}_\pm(p) = \frac{\langle l_1 l_2\rangle}{m} \langle l_1^\pm| + \langle l_2^\mp|$

Calculation

Useful Relations

- $\langle i^{\lambda_1}|\,\gamma_\mu\,|j^{\lambda_2}\rangle \;=\delta_{\lambda_1\lambda_2}\,\langle i^{\lambda_1}|\,\gamma_\mu\,|j^{\lambda_2}\rangle$
- $\langle i^{\lambda_1}|\,\gamma_\mu\gamma_\nu\,|j^{\lambda_2}\rangle = \delta_{-\lambda_1\lambda_2}\,\langle i^{\lambda_1}|\,\gamma_\mu\gamma_\nu\,|j^{\lambda_2}\rangle$
- $\langle i^{\lambda_1}|\,\gamma_\mu\gamma_\nu\gamma_\tau\,|j^{\lambda_2}\rangle = \delta_{\lambda_1\lambda_2}\,\langle i^{\lambda_1}|\,\gamma_\mu\gamma_\nu\gamma_\tau\,|j^{\lambda_2}\rangle$

Products like $\langle i^{\lambda_1}|\,\gamma^\mu\,|j^{\lambda_2}\rangle \ldots \langle i^{\lambda_1}|\,\gamma_\mu\gamma_\nu\,|j^{\lambda_2}\rangle$ can be handled using the spinor representation for gluons $\epsilon^{\pm}_{\mu}(k,q) = \frac{\langle q^{\mp}|\gamma_\mu|k^{\mp}\rangle}{\sqrt{2}\langle q^{\mp}||k^{\mp}\rangle}$
$\ldots \langle i^{\lambda_1}|\,\not{\epsilon}_\lambda(i,j)\gamma_\nu\,|j^{\lambda_2}\rangle$
Further simplification with $\not{\epsilon}_\lambda(i,j) = \lambda\frac{\sqrt{2}}{\langle j^{-\lambda}|i^{\lambda}\rangle}\left(|i^{\lambda}\rangle\,\langle j^{\lambda}| + |j^{-\lambda}\rangle\,\langle i^{-\lambda}|\right)$

Dipol Subtraction Formalism

Cross section

$$\sigma = \sigma^B + \sigma^{NLO} + \dots \tag{14}$$

where

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V \tag{15}$$

- $d\sigma^B$ is integrable over the IR region of the phase space
- $d\sigma^R$ and $d\sigma^V$ are separately affected by IR divergencies produced by soft and collinear divgencies

The soft and collinear divergencies cancel in the sum of Eq. (15), but separate numerical integration leads to infinities.

Auxiliary cross section I

Dipole Subtraction Methode

Idea

construct an auxiliary cross section $d\sigma^A$ that has to fulfill two properties

1. $d\sigma^A$ has the same pointwise singular behaviour as $d\sigma^R$
2. $d\sigma^A$ is *analytically* integrable over the one-parton subspaces, that cause the soft and collinear divergencies

$\Rightarrow d\sigma^A$ is subtracted from the real contribution and added back to the virtual contribution

Auxiliary cross section II

$$\sigma^{NLO} = \int_{m+1} [d\sigma^R - d\sigma^A] + \int_m [d\sigma^V + \int_1 d\sigma^A] \tag{16}$$

- $d\sigma^A$ acts as a *local* counterterm for $d\sigma^R$
  - $\rightsquigarrow$ the difference $d\sigma^R - d\sigma^A$ is numerically integrable over the entire $(m+1)$-particle phase space
- $\int_1 d\sigma^A$ explicitly contains all the ϵ-poles that cancel those of the virtual term $d\sigma^V$

Construction of $d\sigma^A$

$d\sigma^A$ is constructed by a sum over so called dipoles

Dipole contributions can be obtained by an effective two-step-process

- using the Born - Level cross section an m-parton configuration is produced and the emitter and spectator are singled out
- the emitter decays into two partons and the spectator is used to balance momentum conservation

$$d\sigma^A = \sum_{dipoles} d\sigma^B \otimes dV_{dipole} \tag{17}$$

where dV_{dipole} describe two parton decays of the emitter.

Construction of $d\sigma^A$

$$d\sigma^A = \sum_{dipoles} d\sigma^B \otimes dV_{dipole} \tag{18}$$

The product structure of Eq. (18) is due to

- factorization of QCD amplitudes on soft and collinear poles
- factorization property of the phase space

Factorization of the Phase Space

- the factorization property of the phase space permits a mapping from the $(m+1)$-parton phase space to an m-parton phase space times a single parton phase space
- this mapping makes dV_{dipole} analytically and in a process independent manner integrable

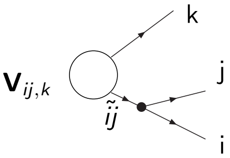
$$\int_{m+1} d\sigma^A = \sum_{dipoles} \int_m d\sigma^B \otimes \int_1 dV_{dipole} = \int_m [d\sigma^B \otimes \mathbf{I}] \qquad (19)$$

where the universal factor $\mathbf{I}$ is defined by

$$\mathbf{I} = \sum_{dipoles} \int_1 dV_{dipole} \qquad (20)$$

(the single parton phase space is process independent)

Dipole Contributions I



In the case of a final-state emitter and a final-state spectator the dipole contribution

$$\mathcal{D}_{ij,k} = d\sigma^B \otimes dV_{ij,k} \tag{21}$$

is given by

$$\mathcal{D}_{ij,k}(p_1, ..., p_{m+1}) = -\frac{1}{(p_i + p_j)^2 - m_{ij}^2} \cdot$$
$$_m < 1, .., \tilde{ij}, .., \tilde{k}, ..| \frac{\mathbf{T}_k \cdot \mathbf{T}_{ij}}{\mathbf{T}_{ij}^2} \mathbf{V}_{ij,k} |1, .., \tilde{ij}, .., \tilde{k}, .. >_m \tag{22}$$

Dipole Contributions II

$$\mathcal{D}_{ij,k}(p_1, ..., p_{m+1}) = -\frac{1}{(p_i + p_j)^2 - m_{ij}^2} \cdot$$
$$_m < 1, .., \tilde{ij}, .., \tilde{k}, ..| \frac{\mathbf{T}_k \cdot \mathbf{T}_{ij}}{\mathbf{T}_{ij}^2} \mathbf{V}_{ij,k} |1, .., \tilde{ij}, .., \tilde{k}, .. >_m \tag{23}$$

with a suitable momentum mapping

$$p_i, p_j, p_k \longrightarrow \tilde{p_{ij}}, \tilde{p_k} \tag{24}$$

that obeys total momentum conservation and the mass-shell conditions

$$\tilde{p_{ij}}^2 = m_{ij}^2, \qquad \tilde{p_k}^2 = m_k^2 \tag{25}$$

Integrated Dipoles

$d\sigma^A$

Construct $d\sigma^A$ as a sum over $\mathcal{D}_{ij,k}(p_1, ..., p_{m+1})$
$\Rightarrow$ $(m+1)$-Parton Phasespace Integration finite

$$\sigma^{NLO} = \int_{m+1} [d\sigma^R - d\sigma^A] + \int_m [d\sigma^V + \int_1 d\sigma^A] \tag{26}$$

analytic integration $\int_1 d\sigma^A$ still needed

Integrated Dipoles

The integrated dipoles are given by

$$\int [dp_i(\tilde{p}_{ij}, \tilde{p}_k)] \mathcal{D}_{ij,k}(p_1, ..., p_{m+1}) = -C_{CF} |\mathcal{M}_m(..., \tilde{p}_{ij}, ..., \tilde{p}_k, ...)|^2 \cdot \\ \cdot \left(\int [dp_i(\tilde{p}_{ij}, \tilde{p}_k)] \frac{\langle \mathbf{V}_{ij,k} \rangle}{(p_i+p_j)^2 - m_{ij}^2} \right) \quad (27)$$

where $\langle \mathbf{V}_{ij,k} \rangle$ is the spin averaged splitting function and C_{CF} contains the colour factors.
This integral can be performed

$$\int [dp_i(\tilde{p}_{ij}, \tilde{p}_k)] \frac{\langle \mathbf{V}_{ij,k} \rangle}{(p_i + p_j)^2 - m_{ij}^2} \equiv \frac{\alpha_S}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{Q^2} \right)^{\epsilon} I_{ij,k}(\epsilon) \quad (28)$$

Phasespace Integration

$$\sigma^{NLO} = \int_{m+1} [d\sigma^R - \sum_{dipoles} \mathcal{D}_{ij,k}] + \int_m [d\sigma^V + d\sigma^B \otimes \mathbf{I}] \qquad (29)$$

Integrationroutine: VEGAS

- adaptive Monte-Carlo Integration
- histogram of the integrand
- adjusting the grid by weighting cells with high contributions of the integrand

Phasespace Integration

$$\sigma = \frac{1}{F} \int \prod_{i=i}^{n} d^4 p_i \delta^+ \left(p_i^2 - m_i^2\right) \sum_{Colour,Spin} |\mathcal{M}|^2 \delta^4 \left(p_{e^-} + p_{e^+} - \sum_{i=1}^{n} p_i \right) \tag{30}$$

with the Fluxfactor $F = 2\lambda^{\frac{1}{2}}(s,0,0)(2\pi)^{(3n-4)}$

massive Threeparton Phasespace

$$R_3 = \int \frac{1}{8} \Theta(1 - (\cos\theta_{12})^2) \Theta((P - k_1 - k_2)^0) dE_1 dE_2 d\Omega_1 d\phi_2 \tag{31}$$

- about 10% of the generated momenta are in the kinematical allowed region

Optimization of Integration

increase the number of kinematically accepted phasespace points

Transformation of Integrationsvariables

$$E_1 = E_1^{'} E_2^{'} + m_t, \qquad (32)$$
$$E_2 = (1 - E_1^{'}) E_2^{'} + m_t.$$

$$\int_{m_t}^{\infty} dE_1 \int_{m_t}^{\infty} dE_2 \mathcal{A}(E_1, E_2) \Theta(E - E_1 - E_2)$$
$$= \int_{m_t}^{E-2m_t} dE_2^{'} \int_0^1 dE_2^{'} E_2^{'} \mathcal{A}(E_1^{'} E_2^{'} + m_t, (1 - E_1^{'}) E_2^{'} + m_t), \qquad (33)$$

Improvement: 10% → 20% of generated points accepted

Summary

Definition of Forward-Backward Asymmetry A_{FB} and order α_S^2 contributions to A_{FB}

Calculation with spinor helicity formalism

Numerical Phasespace Integration using dipole subtraction method