QCD and SUSY-QCD Corrections to Dark Matter Annihilation in the Higgs-Funnel

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[BjHe and M. Klasen, arXiv:0709.0043 [hep-ph], *accepted for publication in Phys. Rev. D*]

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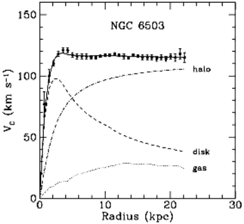
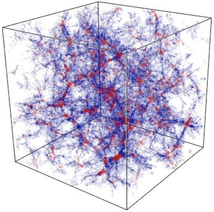
Outline

Outline

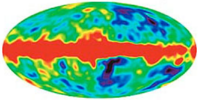
Dark Matter

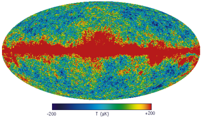
- Astrophysical evidences
  → Rotation curves, structure formation, . . .

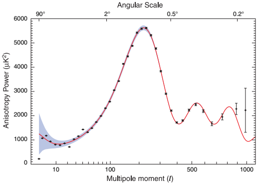




- Cosmological parameters from precision measurements
  → CMB anisotropies, supernovae observations, . . .







Dark Matter

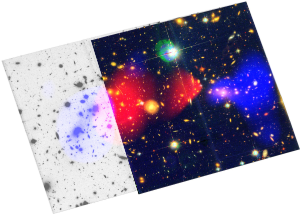
- Cold Dark Matter relic density in our Universe [Hamann *et al.* (2007)]
  → based on WMAP 3-year data, SDSS and SNLS surveys, and BAO data

$$0.094 < \Omega_{\rm CDM} h^2 < 0.136 \qquad \text{(at } 2\sigma\text{)}$$

- First direct proof for existence of Dark Matter in 2006 [Clowe *et al.* (2006)]

- Nature of dark matter still matter of speculation
  → Weakly interacting massive particles (WIMPs)

- New physics provides interesting candidates
  → Supersymmetry, extra dimensions, ...

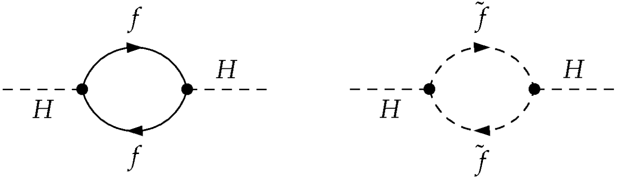


Supersymmetry

- Motivation: Open questions in the Standard Model of Particle Physics
  $\longrightarrow$ Hierarchy problem, unification of gauge couplings, Dark Matter, ...
- Symmetry between bosons and fermions

$$\hat{Q}|f\rangle = |b\rangle \quad \Longleftrightarrow \quad \hat{Q}|b\rangle = |f\rangle$$

  $\longrightarrow$ Superpartners predicted, but not (yet) observed
  $\longrightarrow$ Supersymmetry broken at the electroweak scale
- Can answer (some of) the open questions



Supersymmetry

- Minimal Supersymmetric Standard Model (MSSM)
  $\longrightarrow$ one superpartner for each Standard Model fermion and boson
  $\longrightarrow$ depends on 124 parameters

- Dark Matter candidate: Lightest SUSY particle (LSP)
  $\longrightarrow$ Stable if R-parity conservation
  $\longrightarrow$ Most scenarios: Neutralino $\tilde{\chi}^0_1$ (or gravitino $\tilde{G}$)

$$\tilde{\chi}^0_i \;=\; N_{1i}\tilde{H}_1 + N_{2i}\tilde{H}_2 + N_{3i}\tilde{B} + N_{4i}\tilde{W}$$

- Constraints on large parameter space mainly from collider experiments
  $\longrightarrow$ Limits on SUSY mass parameters
  $\longrightarrow$ Additional information from cosmological measurements

Relic Density Calculation

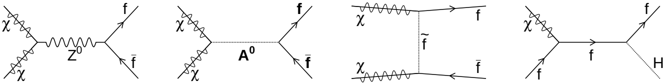
- Neutralino relic density can be computed for a given SUSY scenario

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\rm ann} v \rangle \left(n^2 - n_{\rm eq}^2 \right) \quad \Longrightarrow \quad \Omega_{\rm CDM} h^2 \propto \frac{1}{\langle \sigma_{\rm ann} v \rangle}$$

$\longrightarrow$ Identify (dis)favoured regions with respect to WMAP 2σ range

$$0.094 < \Omega_{\rm CDM} h^2 < 0.136$$

- Cross section $\sigma_{\rm ann}$ involves all (co)annihilation processes



- Public codes performing relic density calculation

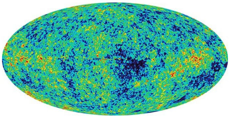
$\longrightarrow$ DarkSUSY [Gondolo *et al.* (2003)]

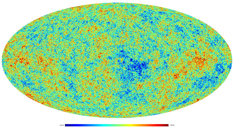
$\longrightarrow$ micrOMEGAs [Bélanger *et al.* (2005)]

Relic Density Calculation

- All (co)annihilation processes implemented in public codes
  $\longrightarrow$ no (or not full) QCD corrections
  $\longrightarrow$ no SUSY-QCD corrections

- New cosmological experiments in a near future (Planck 2008)
  $\longrightarrow$ Better experimental precision for $\Omega_{\rm CDM} h^2$
  $\longrightarrow$ More accurate prediction of $\sigma_{\rm ann}$ needed
  $\longrightarrow$ Radiative corrections to neutralino annihilation become important



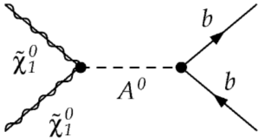


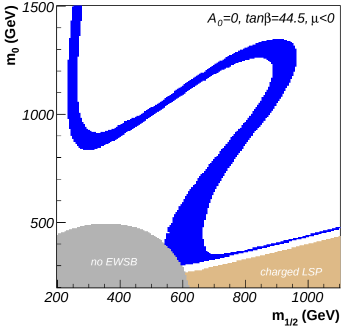
Higgs-Funnel in mSUGRA

- Minimal Supergravity (mSUGRA) scenario
  $\longrightarrow$ Five universal parameters at high scale

  $m_0, \quad m_{1/2}, \quad A_0, \quad \tan\beta, \quad \mathrm{sgn}(\mu)$

- Higgs-Funnel region in m_0-$m_{1/2}$ plane
  $\longrightarrow$ Annihilation through pseudoscalar Higgs exchange dominates





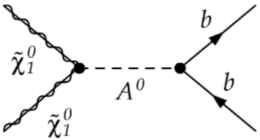
- Large $\tan\beta$ favoured by theory
  $\longrightarrow$ Yukawa coupling unification in GUT theories [Carena *et al.* (1994)]
- Process claimed to be compatible with gamma-ray excess observed by EGRET [de Boer *et al.* (2005)]

Outline

Leading Order Calculation

- Anti-symmetrization of Majorana initial state

$$\frac{1}{\sqrt{2}}\left[u(p_a)\bar{v}(p_b) - u(p_b)\bar{v}(p_a)\right]$$



- Leading order annihilation cross section
  → Process important at large $\tan\beta$

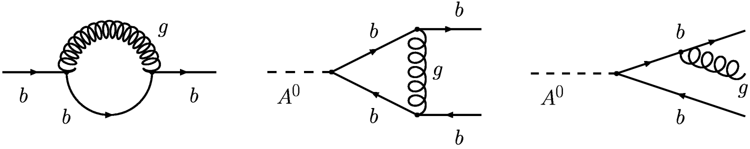
$$\sigma_{\rm LO} v = \frac{1}{2}\frac{\beta_b}{8\pi s}\frac{N_C g^2 T_{\rm A11}^2 h_{\rm Abb}^2 s^2}{|s - m_A^2 + i m_A \Gamma_A|^2} \propto m_b^2 \tan^2\beta$$

- Non-relativistic expansion agrees with previous results [Jungman *et al.* (1996)]

$$s \doteq 4m_\chi^2\left(1 + \frac{v^2}{4}\right) + \mathcal{O}(v^4) \quad \Longrightarrow \quad \sigma_{\rm LO} v = a + bv^2 + \mathcal{O}(v^4)$$

QCD Corrections at $\mathcal{O}(\alpha_s)$

- Gluon loops in quark self-energy and vertex correction
  $\longrightarrow$ On-shell renormalization to cancel UV-singularities



- Remaining IR-singularities vanish with real gluon emission
  $\longrightarrow$ Dipole subtraction method [Catani *et al.* (2002)]

$$\sigma_{\rm QCD} = \left[\sigma_{\rm V} + \int d\sigma_{\rm aux}\right] + \int \left[d\sigma_{\rm R} - d\sigma_{\rm aux}\right]$$

- Annihilation cross section including $\mathcal{O}(\alpha_s)$ QCD corrections

$$\sigma_{\rm NLO} = \sigma_{\rm LO}\left[1 + \Delta_{\rm QCD}\right]$$

QCD Corrections at $\mathcal{O}(\alpha_s)$

- Logarithmic mass singularity at high energies ($m_b^2 \ll s$, $\beta_b \to 1$)

$$\Delta_{\rm QCD}^{\rm (HE)} \simeq \frac{\alpha_s C_F}{\pi} \left[-\frac{3}{2} \log \frac{s}{m_b^2} + \frac{9}{4} \right]$$

$\longrightarrow$ Resummation to all orders using renormalization group

$\longrightarrow$ Use running quark mass in Yukawa coupling [Braaten *et al.* (1980)

$$h_{\rm Abb} = -\frac{g m_b \tan\beta}{2 m_W} \longrightarrow -\frac{g \bar{m}_b(Q^2) \tan\beta}{2 m_W}$$

]

- Interpolation between low and high energy regimes

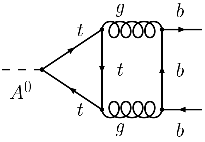
$$\Delta_{\rm QCD} = \frac{4 m_b^2}{s} \Delta_{\rm QCD}^{\rm (LE)} + \left(1 - \frac{4 m_b^2}{s} \right) \Delta_{\rm QCD}^{\rm (HE)}$$

Higher Order QCD Corrections

- Remaining finite QCD-corrections are known up to $\mathcal{O}(\alpha_s^3)$ [Chetyrkin (1997)]

$$\Delta_{\rm QCD} = \frac{\alpha_s(s)}{\pi} C_F \frac{17}{4} + \frac{\alpha_s^2(s)}{\pi^2}\left[35.94 - 1.36 n_f\right] + \frac{\alpha_s^3(s)}{\pi^3}\left[164.14 - 25.76 n_f + 0.259 n_f^2\right]$$

- Top-quark loop correction [Chetyrkin *et al.* (1996)]
  $\longrightarrow$ largely suppressed in the Higgs-Funnel region



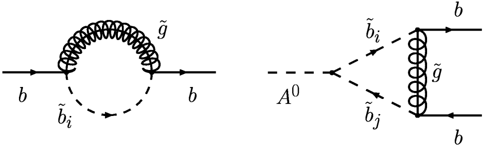
$$\Delta_{\rm top} = \frac{1}{\tan^2\beta}\frac{\alpha_s^2(s)}{\pi^2}\left[\frac{23}{6} - \log\frac{s}{m_t^2} + \frac{1}{6}\log^2\frac{\bar{m}_b^2(s)}{s}\right]$$

- QCD up to $\mathcal{O}(\alpha_s^2)$ included in `DarkSUSY 4.1` [Gondolo *et al.* (2004)]

$$\sigma_{\rm NLO} = \sigma_{\rm LO}\left[1 + \Delta_{\rm QCD} + \Delta_{\rm top}\right]$$

SUSY-QCD Corrections at $\mathcal{O}(\alpha_s)$

- Gluino loops in quark self-energy and gluino vertex correction



- Self-energy leads to mass renormalization

$$\frac{\Delta m_b}{m_b} = \frac{\alpha_s(s)}{4\pi} C_F \left[B_0(m_b^2; m_{\tilde{b}_1}^2, m_{\tilde{g}}^2) - B_0(m_b^2; m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) \right] \sin 2\theta_{\tilde{b}}$$
$$+ \frac{\alpha_s(s)}{4\pi} \frac{C_F}{m_b^2} \sum_{i=1,2} \left[A_0(m_{\tilde{g}}^2) + A_0(m_{\tilde{b}_i}^2) + (m_{\tilde{b}_i}^2 - m_{\tilde{g}}^2 - m_b^2) B_0(m_b^2; m_{\tilde{g}}^2, m_{\tilde{b}_i}^2) \right]$$

- Limit of high SUSY masses: $m_b \ll m_{\tilde{b}_i}, m_{\tilde{g}}$

$$\frac{\Delta m_b}{m_b} \propto \sin 2\theta_{\tilde{b}} = \frac{2 m_b}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} \left[A_b - \mu \tan\beta \right]$$
$$\frac{\Delta m_b}{m_b} = \frac{\alpha_s(s)}{\pi} C_F \frac{m_{\tilde{g}}}{2} \left[A_b - \mu \tan\beta \right] C_0(0,0,0; m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2)$$

SUSY-QCD Corrections at $\mathcal{O}(\alpha_s)$

- Vertex correction equals mass renormalization up to a factor $1/\tan^2\beta$
- Total SUSY correction in "low energy" limit and for $A_b \ll \mu\tan\beta$

$$\Delta^{(\mathrm{LE})}_{\mathrm{SUSY}} = \frac{\alpha_s(s)}{\pi} C_F \left(1 + \frac{1}{\tan^2\beta}\right) m_{\tilde{g}}^2 \mu \tan\beta \, C_0(0,0,0; m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}})$$

- Δm_b significant for large $\tan\beta$ [Carena *et al.* (2000)]
  $\longrightarrow$ Resummation of its contribution in the Yukawa coupling

$$m_b \longrightarrow \frac{m_b}{1 + \lim_{A_b \to 0} \frac{\Delta m_b}{m_b}}$$

- A_b might be of same order of magnitude as $\mu\tan\beta$
  $\longrightarrow$ depending on SUSY scenario
  $\longrightarrow$ Additional resummation of this contribution [Guasch *et al.* (2003)]

$$\lim_{A_b \to 0} \frac{\Delta m_b}{m_b} \longrightarrow \frac{\lim_{A_b \to 0} \frac{\Delta m_b}{m_b}}{1 + \lim_{\mu\tan\beta \to 0} \frac{\Delta m_b}{m_b}}$$

Final cross section result

- Final annihilation cross section including QCD and SUSY-QCD corrections

$$\sigma_{\rm NLO} = \sigma_{\rm LO}(s)\Big[1 + \Delta_{\rm QCD} + \Delta_{\rm top} + \Delta_{\rm SUSY}\Big]$$

- $\sigma_{\rm LO}(s)$ comprises QCD and SUSY-QCD mass resummation
  $\rightarrow$ running quark mass in Yukawa coupling

- Finite remainders

$$\Delta_{\rm QCD} = \Delta^{(1)}_{\rm QCD}(s) + \frac{\alpha_s^2(s)}{\pi^2}\big[35.94 - 1.36 n_f\big] + \frac{\alpha_s^3(s)}{\pi^3}\big[164.14 - 25.76 n_f + 0.259 n_f^2\big]$$

$$\Delta_{\rm top} = \frac{1}{\tan^2\beta}\frac{\alpha_s^2(s)}{\pi^2}\left[\frac{23}{6} - \log\frac{s}{m_t^2} + \frac{1}{6}\log^2\frac{\bar{m}_b^2(s)}{s}\right]$$

$$\Delta_{\rm SUSY} = \frac{\alpha_s(s)}{\pi} C_F \frac{1+t_\beta^2}{t_\beta} m_{\tilde{g}}\,\mu\left[C_0(m_b^2, s, m_b^2; m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) - C_0(0,0,0; m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2)\right]$$

- Agrees with previous results [Dabelstein (1995), Coarasa *et al.* (1996), Guasch *et al.* (2003)]

Outline

Model and Method

- Five mSUGRA parameters at GUT scale

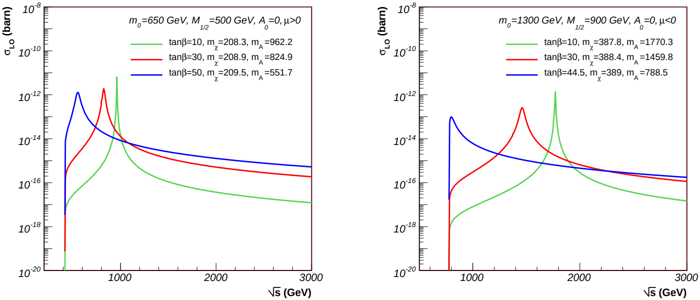
$$m_0, \quad m_{1/2}, \quad A_0, \quad \tan\beta, \quad \mathrm{sgn}(\mu)$$

- Mass spectrum at electroweak scale
  $\longrightarrow$ SPheno (RGE running) [Porod (2003)]
  $\longrightarrow$ FeynHiggs (full spectrum) [Heinemeyer *et al.* (1998, 2002)]
- Cold dark matter relic density
  $\longrightarrow$ DarkSUSY 4.1 [Gondolo *et al.* (2004)]
  $\longrightarrow$ include full QCD and SUSY-QCD corrections to $\tilde{\chi}\tilde{\chi} \rightarrow A^0 \rightarrow b\bar{b}$
- Standard Model parameters [Yao *et al.* , Particle Data Group (2006)]

$$m_b(m_b) = 4.2 \text{ GeV}, \qquad m_t = 174.2 \text{ GeV}$$

Leading Order Cross Section

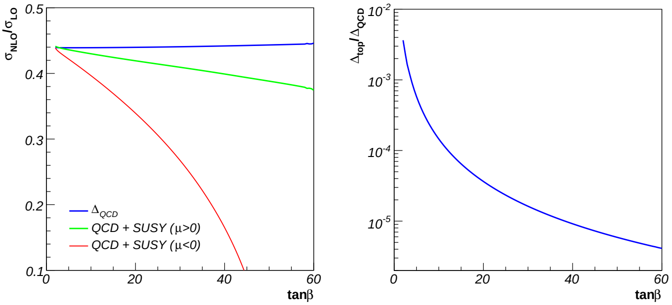
- Leading order cross section as a function of center-of-momentum energy $\sqrt{s}$



- Process important at large $\tan\beta$ near threshold

Annihilation Cross Section at NLO

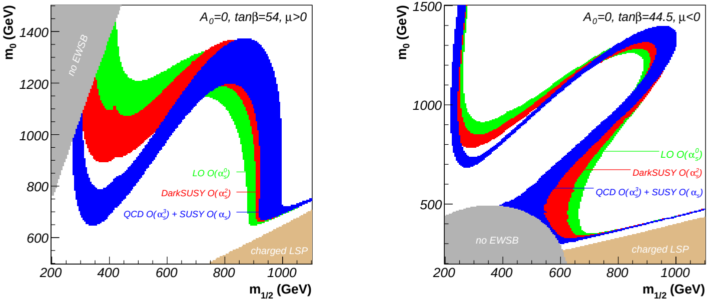
- QCD correction decreases cross section by about 50%



- SUSY-QCD correction contributes up to another 40%
  $\longrightarrow$ depending on sgn(μ) and tan β
- Top-quark loop contribution negligible for large tan β

Effect on Relic Density

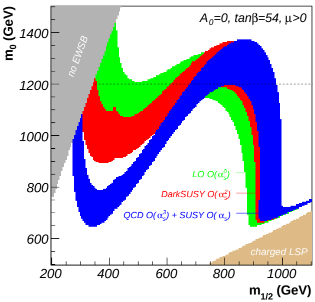
- Identify WMAP favoured regions including different (SUSY-)QCD corrections

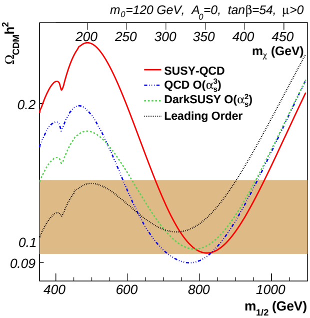


- Smaller annihilation cross section compensated by smaller SUSY masses
- Effect reversed on the Higgs pole, where Higgs width is important

Effect on Relic Density

- Inspect relic density as a function of $m_{1/2}$ for fixed m_0, A_0, $\tan\beta$, $\mathrm{sgn}(\mu)$

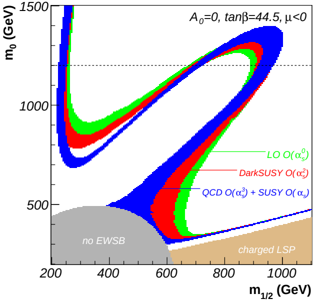


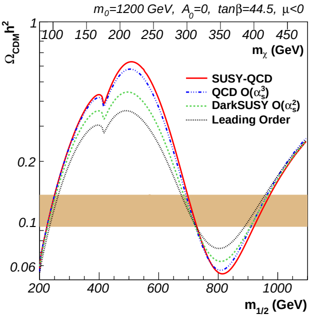


- Effect of $\mathcal{O}(\alpha_s^2)$ QCD corrections is considerably enhanced by $\mathcal{O}(\alpha_s^3)$ QCD and $\mathcal{O}(\alpha_s)$ SUSY-QCD corrections
- Local minimum at $m_{1/2} \approx 420$ GeV corresponds to $m_\chi = m_t$

Effect on Relic Density

- Inspect relic density as a function of $m_{1/2}$ for fixed m_0, A_0, $\tan\beta$, $\text{sgn}(\mu)$





- Effect less important than for $\mu > 0$

Impact of Corrections to the Higgs Width

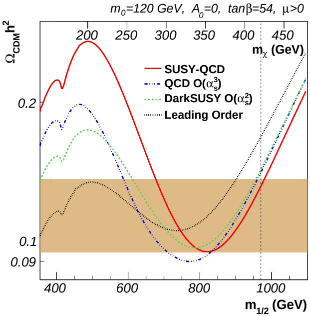
- Correction diagrams contribute also to Higgs decay width
  $\longrightarrow$ particularly important on the pole $m_A = 2m_\chi$

$$\Gamma^{(\mathrm{NLO})}_{\mathrm{A}\to b\bar{b}} = \Gamma^{(\mathrm{LO})}_{\mathrm{A}\to b\bar{b}}(s)\left[1 + \Delta_{\mathrm{QCD}} + \Delta_{\mathrm{top}} + \Delta_{\mathrm{SUSY}}\right]$$

- Correction terms influence distance between resonance and pole

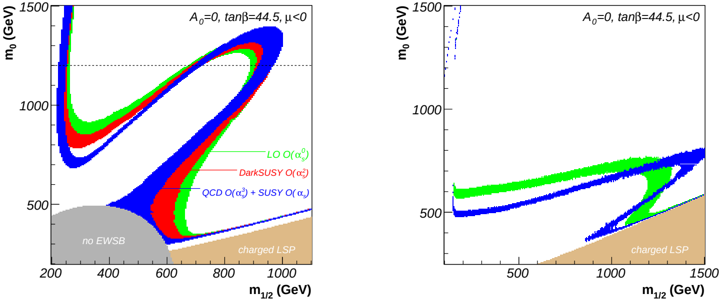
$$\sigma_{\mathrm{NLO}} \propto \frac{1}{|s - m_{\mathrm{A}}^2 + i m_{\mathrm{A}} \Gamma_{\mathrm{A}}|^2}$$

- For smaller width minimum of relic density approaches the Higgs pole
  $\longrightarrow$ Correction effect reversed around pole



Dependence on spectrum generator

- `SPheno/FeynHiggs` vs. `ISAJET`



- Neutralino and Higgs masses especially sensitive to spectrum code [Allanach *et al.* (2003), Bélanger *et al.* (2005)]
- Effect of (SUSY-)QCD corrections remains the same

Outline

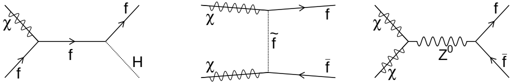
Summary

- Relic density calculation is an interesting tool to constrain supersymmetry
  $\longrightarrow$ in times of high-precision cosmology radiative corrections are important
- Calculation of full QCD and SUSY-QCD corrections to $\tilde{\chi}\tilde{\chi} \rightarrow A^0 \rightarrow b\bar{b}$
  $\longrightarrow$ dominant process in the Higgs funnel region [BjHe and M. Klasen (2007)]
- Numerical evaluation of annihilation cross section and relic density
  $\longrightarrow$ important decrease of annihilation cross section
  $\longrightarrow$ favoured relic density contour shifted to smaller masses
  $\longrightarrow$ effect of corrected Higgs width around pole
- Corrections strongly influence extraction of SUSY mass parameters
  $\longrightarrow$ to be included in common analysis tools like DarkSUSY

Outlook

- SUSY-QCD Corrections to neutralino-squark coannihilation

  [Freitas (2007)]



- SUSY-QCD Corrections for remaining annihilation processes
  $\longrightarrow$ no numerical results yet [BjHe, K. Kovarik, and M. Klasen, *in preparation*]

- Include all SUSY-QCD Corrections into common analysis tools
  $\longrightarrow$ provide complete consistent relic density calculation code

- Compare DarkSUSY and micrOMEGAs
  $\longrightarrow$ differences between the two codes in *A-funnel* region