NNLO Virtual QCD Corrections for W Pair Production at the LHC

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Outline

- Introduction
- W pair production important for LHC
- Motivation for studying q q $\rightarrow$ W W
- Importance of high precision – adding higher orders of the perturbative calculation with mass dependence
- Historical review of the calculations on q q $\rightarrow$ W W
- NNLO Virtual Corrections: Technical details
- Mellin-Barnes representations of Feynman Integrals
- Results
- Outlook

Soon (May 2008?) in the LHC era

LHC: an experimantal purgatory

Collision energy: 14 TeV
Luminosity: 10 fb^{-1} per year in the first stage

- VIP: Higgs ?
- Consistency of SM ?

- SUSY ?
- Extra Dimensions ?

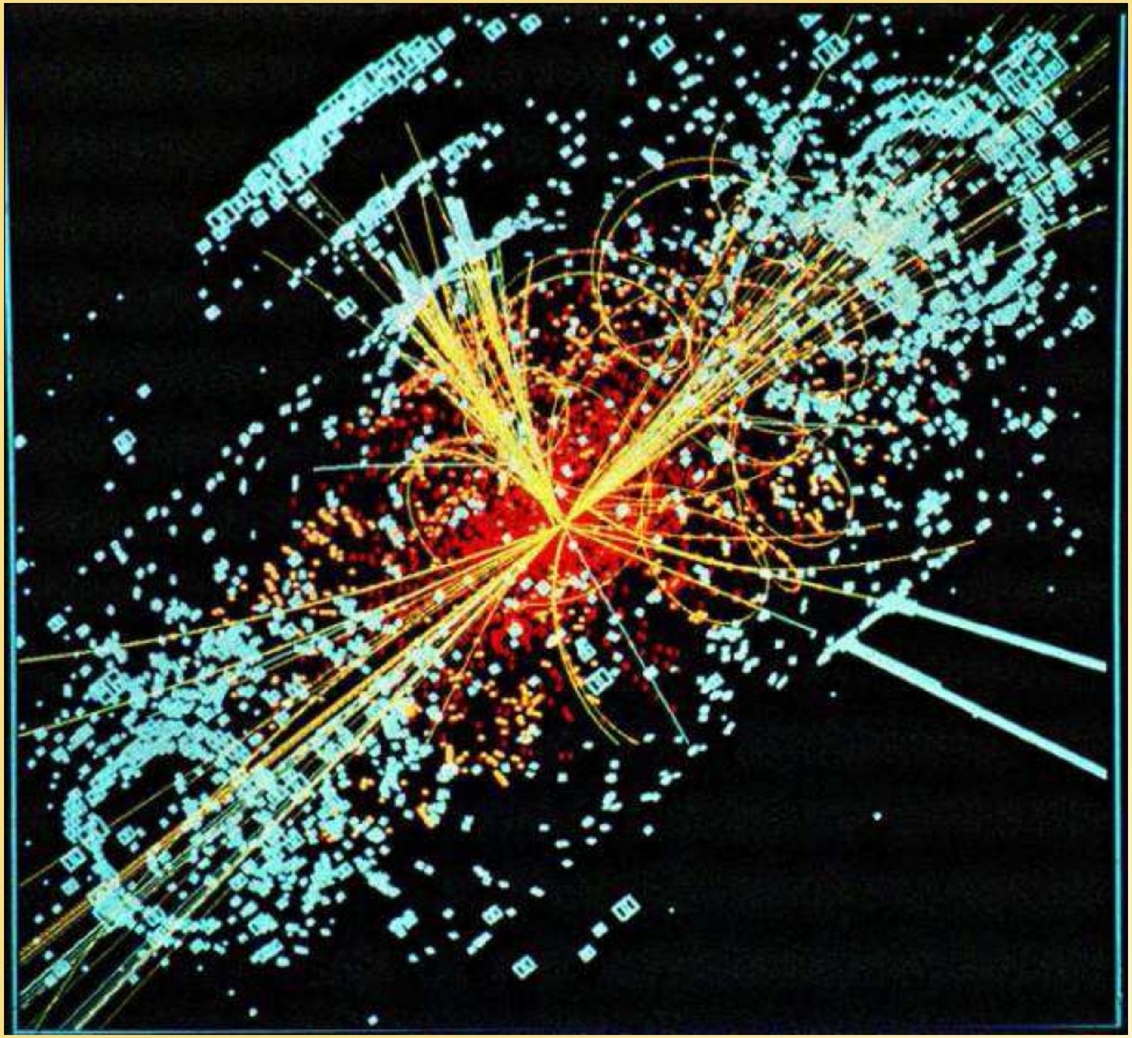
The elusive Higgs boson

Higgs:

•Only constituent of the SM not experimentally observed yet.
•Electroweak symmetry breaking
•Description of particle masses

Discovery by itself is not enough!
Properties of the Higgs needed to exclude or verify alternative models

In the LHC era life doesn't appear to be simple...



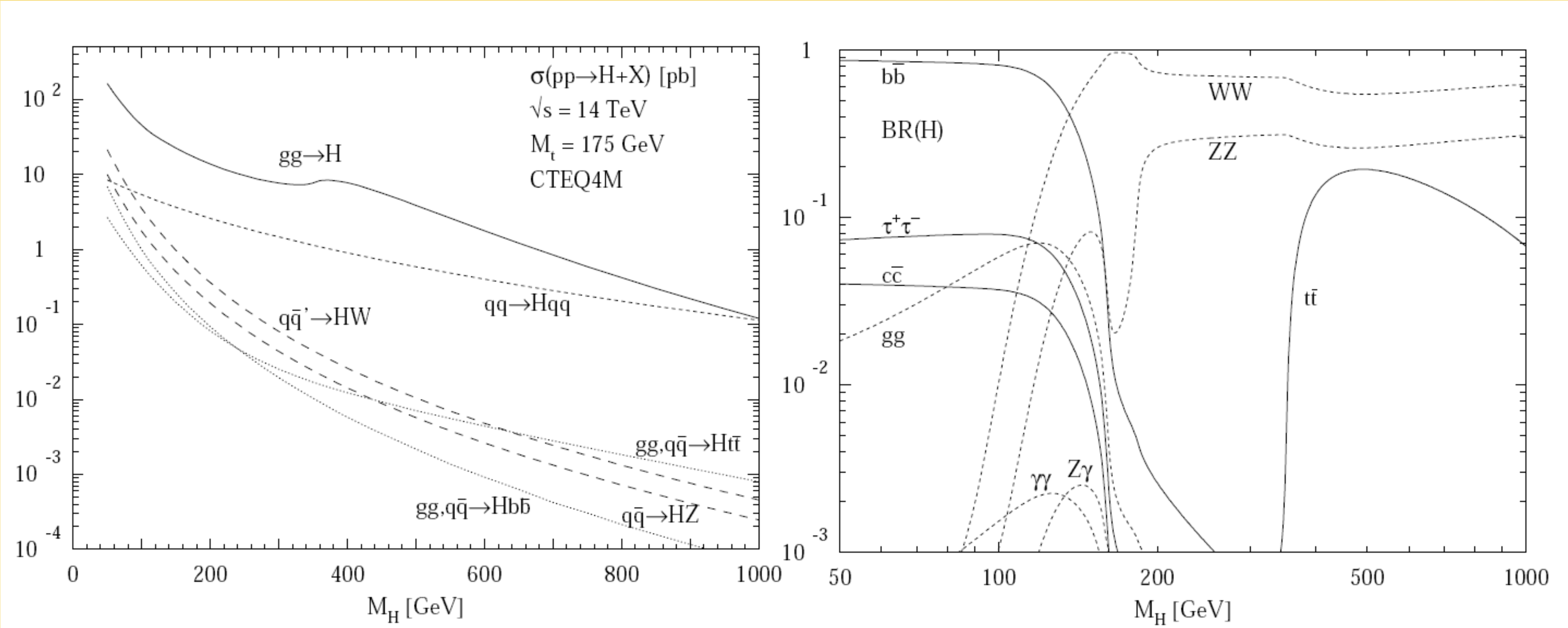
A simulated event of Higgs boson production in the CMS detector

Going after the Higgs

- Pick up your signal process: an observable characteristic of Higgs production
- Try to avoid or suppress the background
- Have accurate predictions for both signal and background processes from the theory point of view.

LHC has the energy and luminosity required to discover Higgs over the whole allowed range
114 GeV < M_H < 700 GeV

LHC Higgs production ...

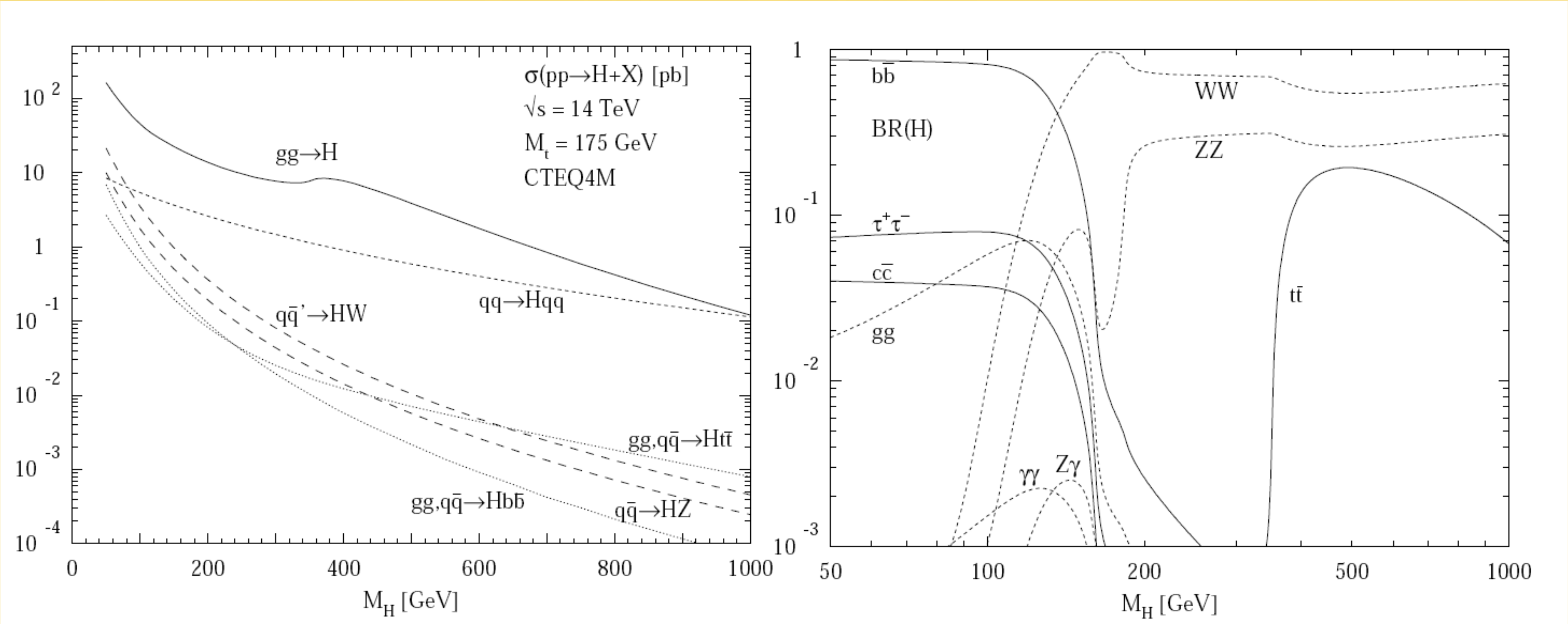


spira 1997

Gluon Fusion channel is the dominant production mechanism up to M_H ~ 1 TeV : g g → H

Sub-dominant production process is Weak Boson Fusion: q q → V V → q q H

LHC Higgs production and decay



spira 1997

Once the Higgs is produced it will eventually decay into different particles depending on its mass.
In the Higgs mass range 140 – 180 GeV the main decay mode is into W W

Main discovery Channels

•M_H: 114 – 140 GeV

$H \rightarrow \gamma\,\gamma$

•M_H: 140 – 180 Gev

$H \rightarrow W\,W \rightarrow 2\,l$ + missing Energy E_T

•M_H: 180 – 600 Gev

$H \rightarrow Z\,Z \rightarrow 4\,l$

Motivation for high accuracy in W pair Production

W pair production important:

- **<u>as a signal</u>**
  Accurate knowledge needed to disentangle “new Physics”
  Testing ground for non-Abelian structure of the SM

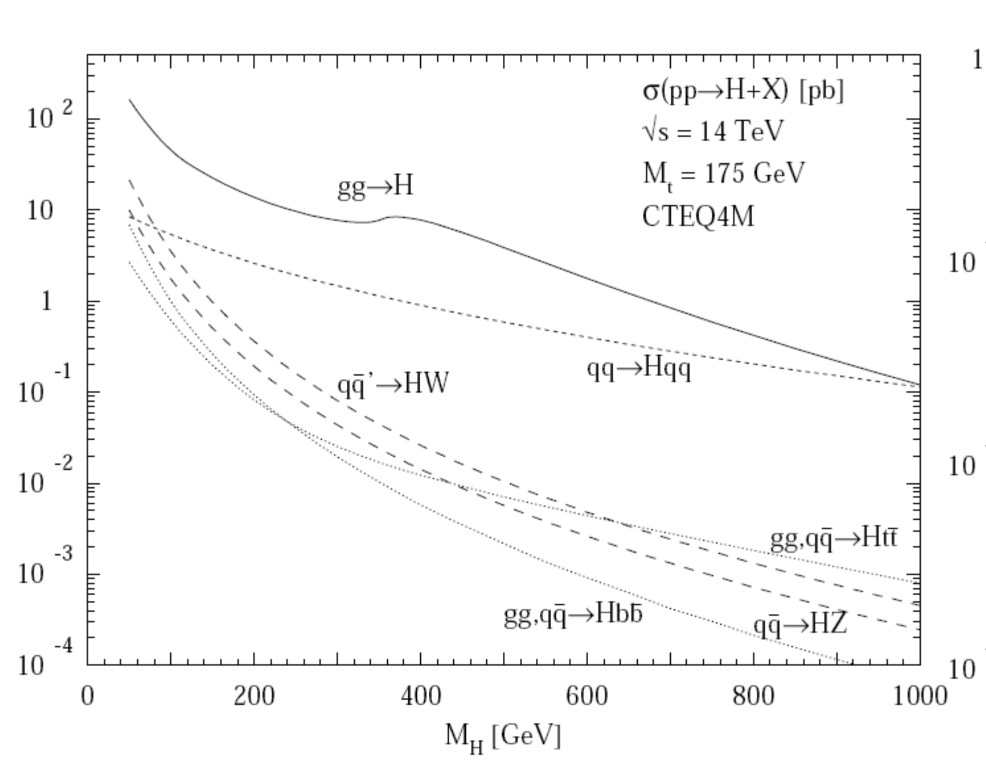
Motivation for high accuracy in W pair Production

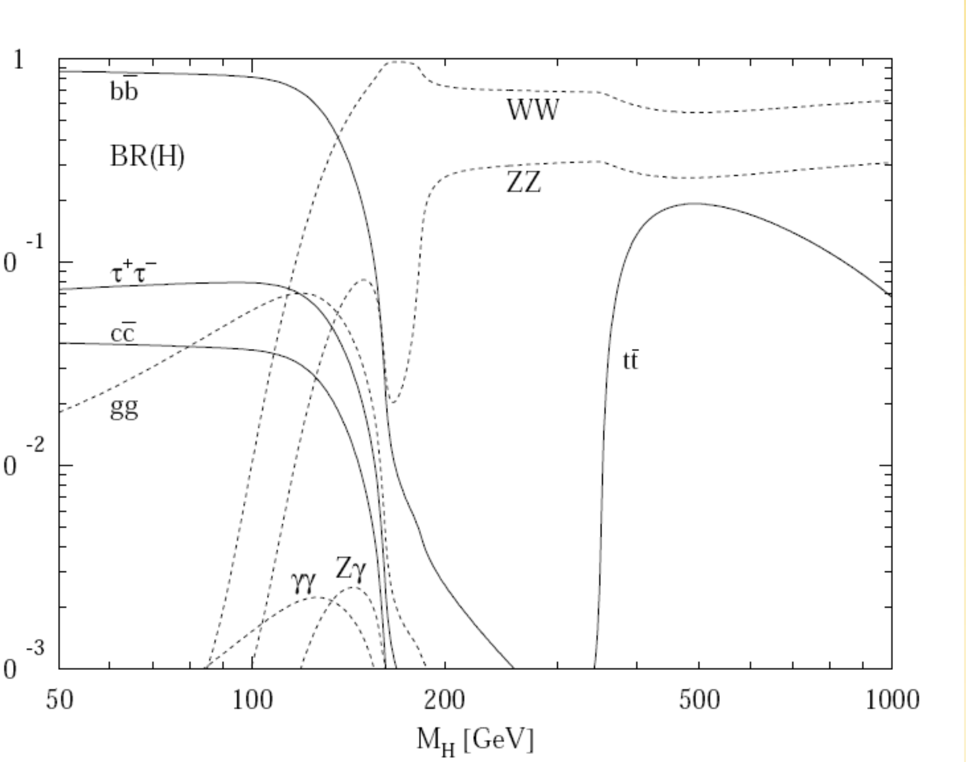
W pair production important:

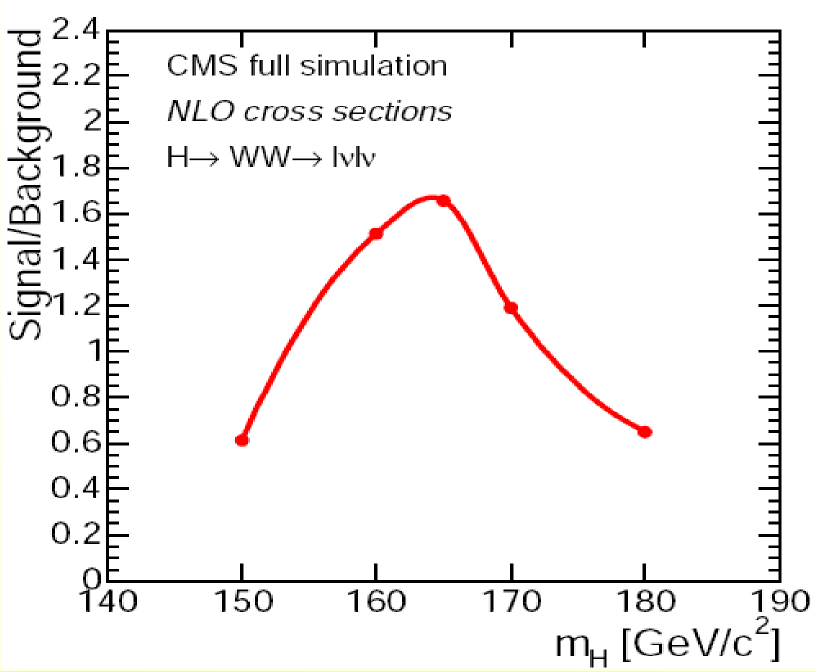
- **<u>as important (irreducible) background</u>** to the Higgs boson discovery channel:

$$pp \longrightarrow H \longrightarrow WW \longrightarrow \text{leptons}$$

Signal – background ratio







Need for higher order corrections

Rule of thumb*

In general:

LO: The first order term of the perturbative expansion gives an order of magnitude estimate

NLO: Second order brings into the game 10-30 % corrections and usually a good quantitative description

NNLO: Precision of few percent level

* Kunszt

State of the art for Higgs production and Higgs to W's decay

QCD corrections to g g → H

NLO: Contribute ~ 70%

Djouadi,Graudenz,Spira,Zerwas; Dawson

NNLO: Contribute an additional 20% for LHC

Harlander, Kilgore;Anastasiou, Melnikov;

Ravindran, Smith, van Neerven

With a Jet veto at NNLO: corrections ~ 85%

Catani, de Florian, Grazzini;

Davatz, Dissertori, Dittmar, Grazzini, Pauss

Anastasiou, Melnikov, Petrielo

NNLO **$H \to W\,W \to l\,\nu\,l\,\nu$**

Anastasiou, Dissertori, Stöckli

- qq→WW

- loop induced gg→WW

- $q\bar{q} \to WW$

Receives a 70% enhancement at NLO with no cuts. With a jet veto the enhancements fall to 20-30%

Dixon, Kunszt, Signer

- loop induced $gg \to WW$

- qq→WW
  Receives a 70% enhancement at NLO with no cuts. With a jet veto the enhancements fall to 20-30%

  Dixon, Kunszt, Signer

- loop induced gg→WW
  Formally a NNLO process. Contributes to the quark annihilation channel at $\mathcal{O}(\alpha_s^2)$.
  Enhanced by the large gluon flux.
  After Higgs search cuts it increases the background by 30%, with no cuts by 5%

  Binoth, Ciccolini, Kauer, Krämer; Duhrssen et al

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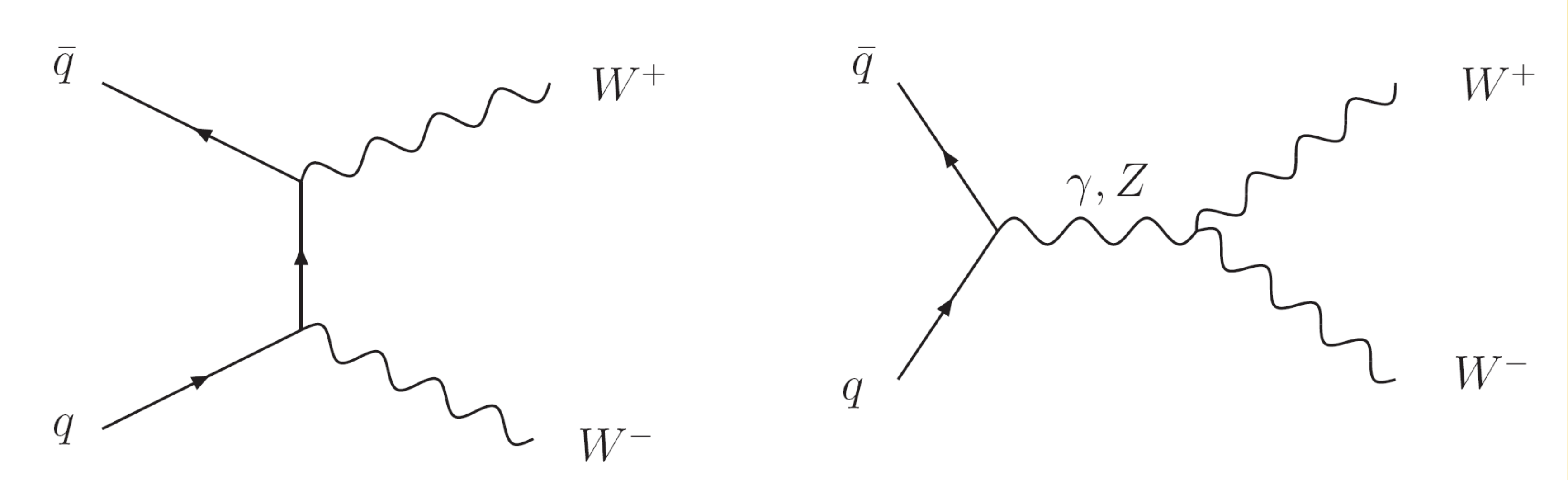
Binoth, Ciccolini, Kauer, Krämer; Duhrssen et al

Necessity of NNLO Calculation for % level accuracy

History I (LO)



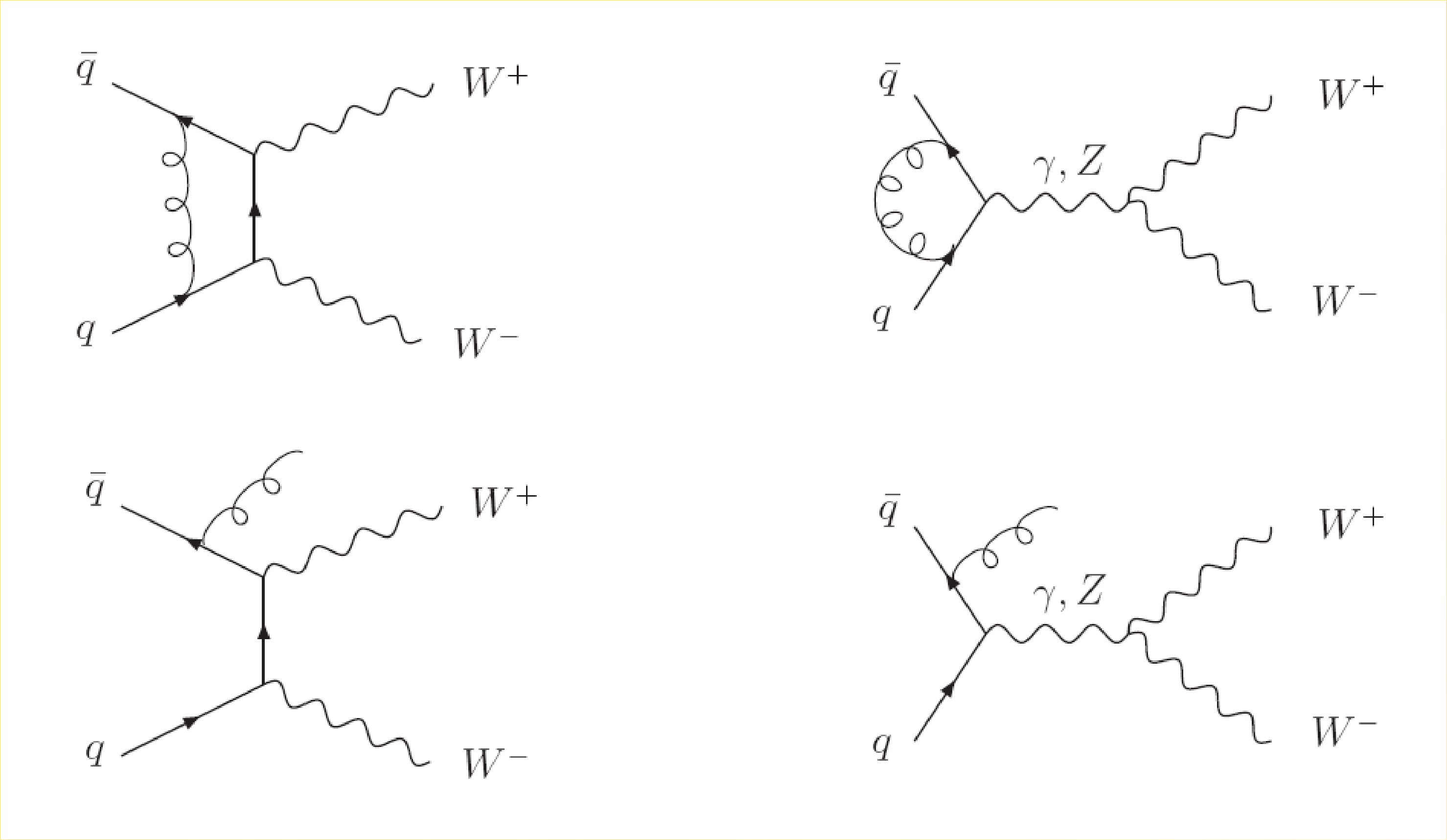
LO Calculation

Brown, Mikaelian (1979)

CERN Discovery of Z and W bosons

(1983)

History II (NLO)



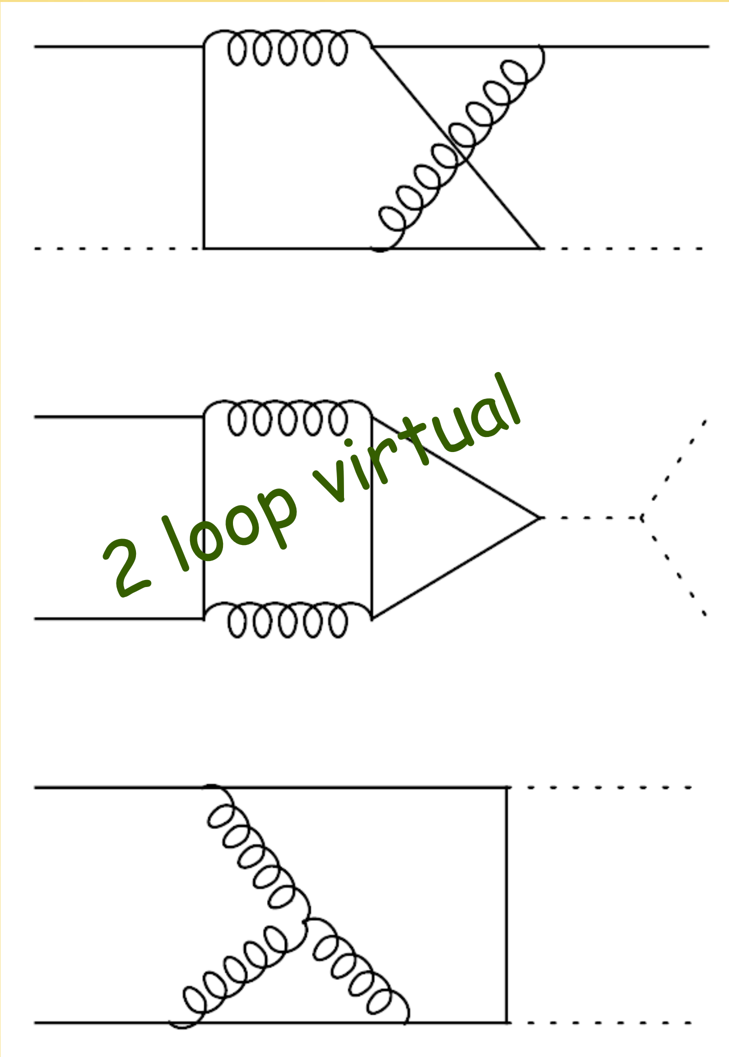
NLO Calculation Ohnemus (1991); Frixione (1993)

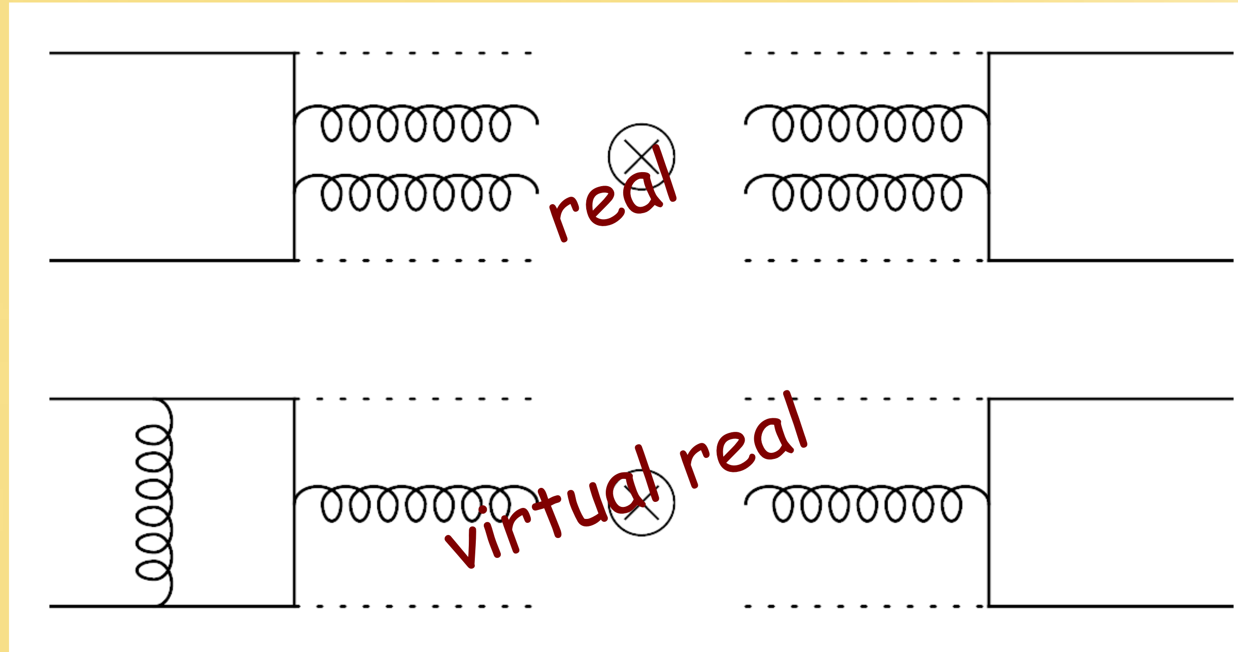
Also, Ohnemus(1994)

Dixon, Kunszt, Signer (1998,1999)

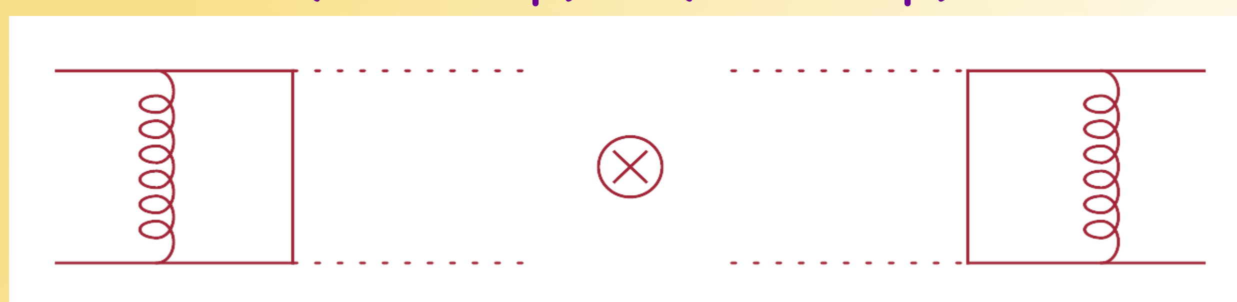
Campbell, K. Ellis (1999)

Present: NNLO Corrections

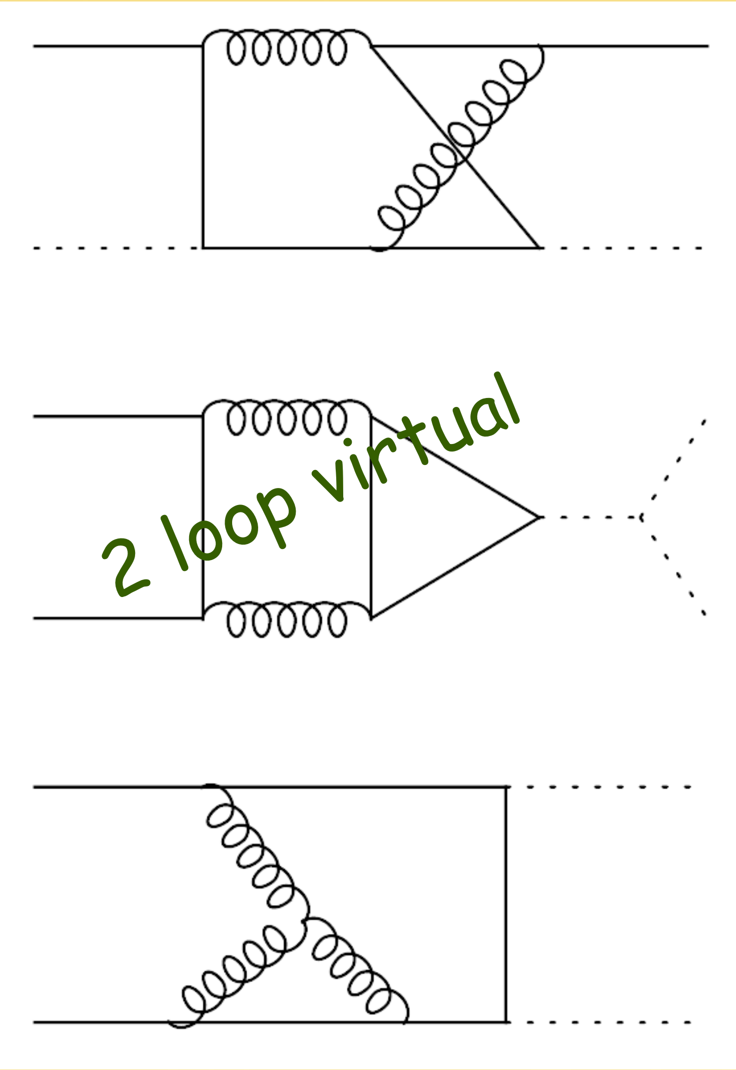


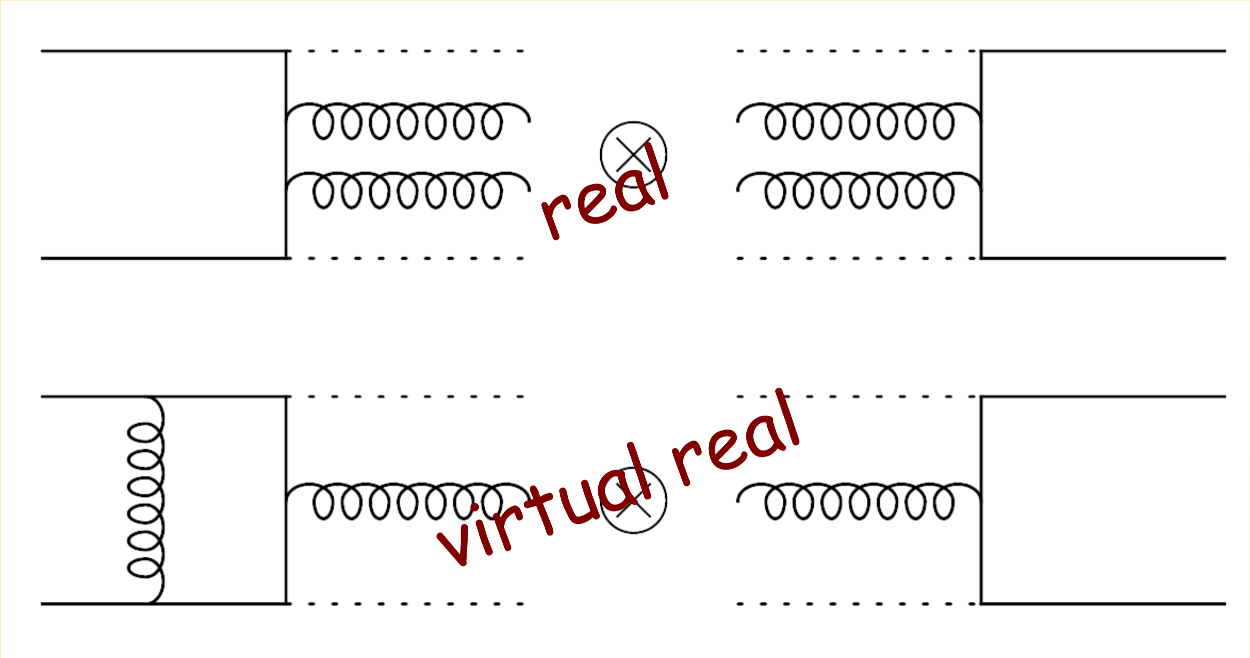


(one loop) ⊗ (one loop)*

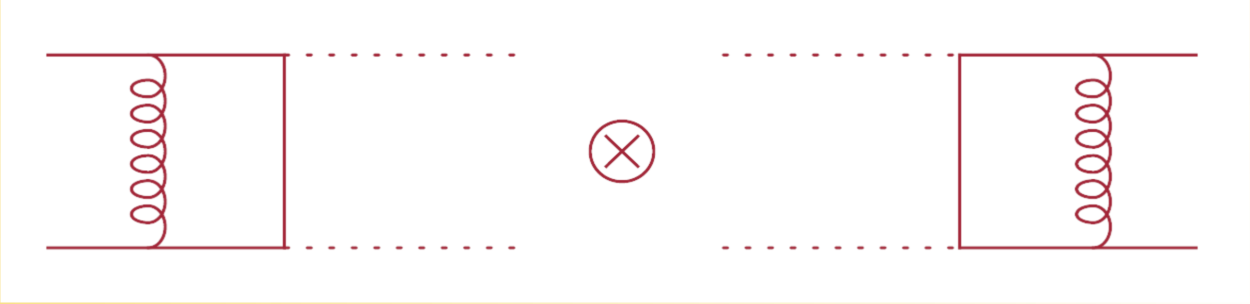


Present: NNLO Corrections



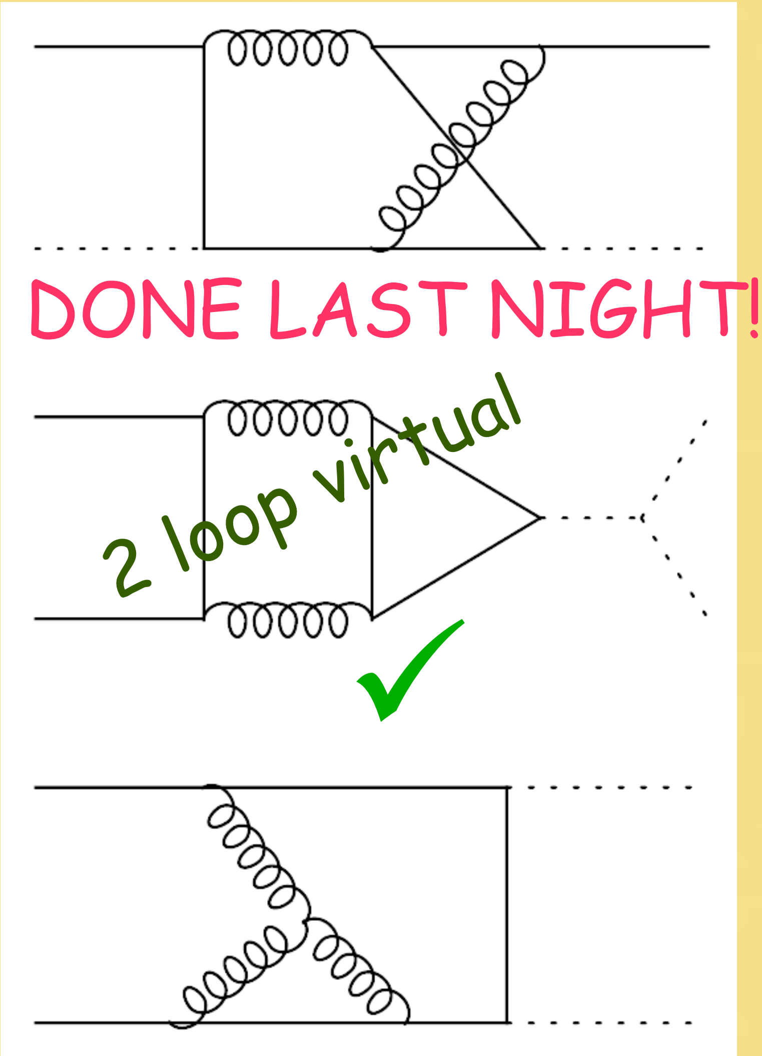


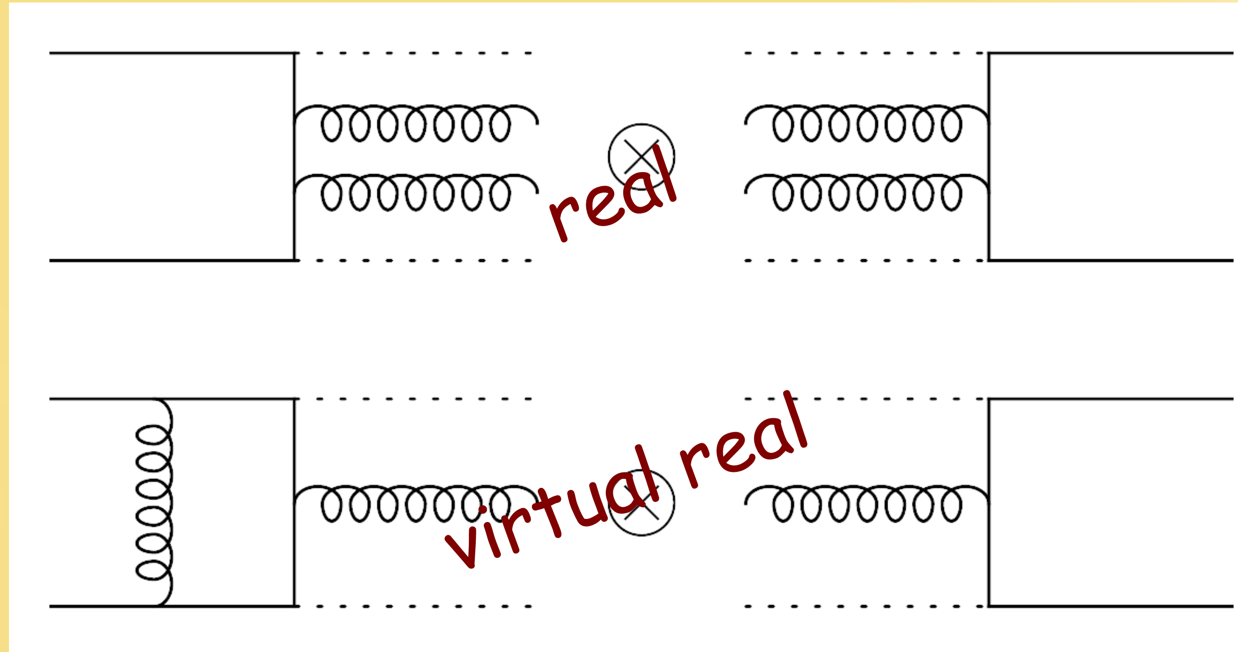
(one loop) ⊗ (one loop)*



DONE ✓ ➡

Present: NNLO Corrections





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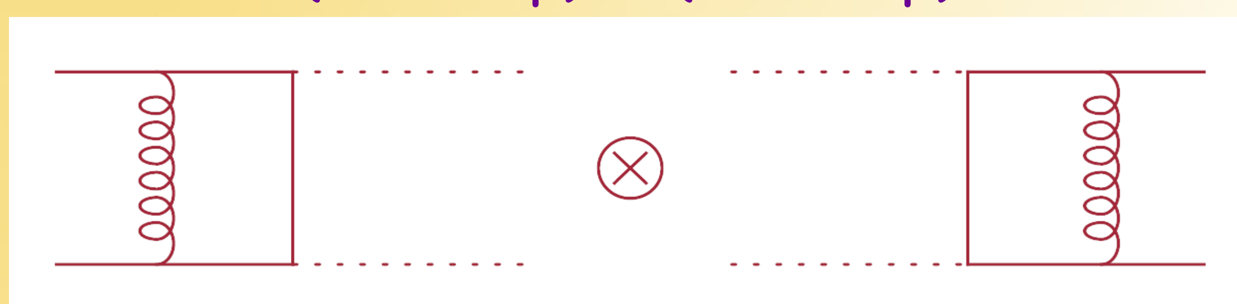


DONE ✓

Contributions to the cross section

$$d\sigma_n = d\Phi_n \,|M_n|^2$$

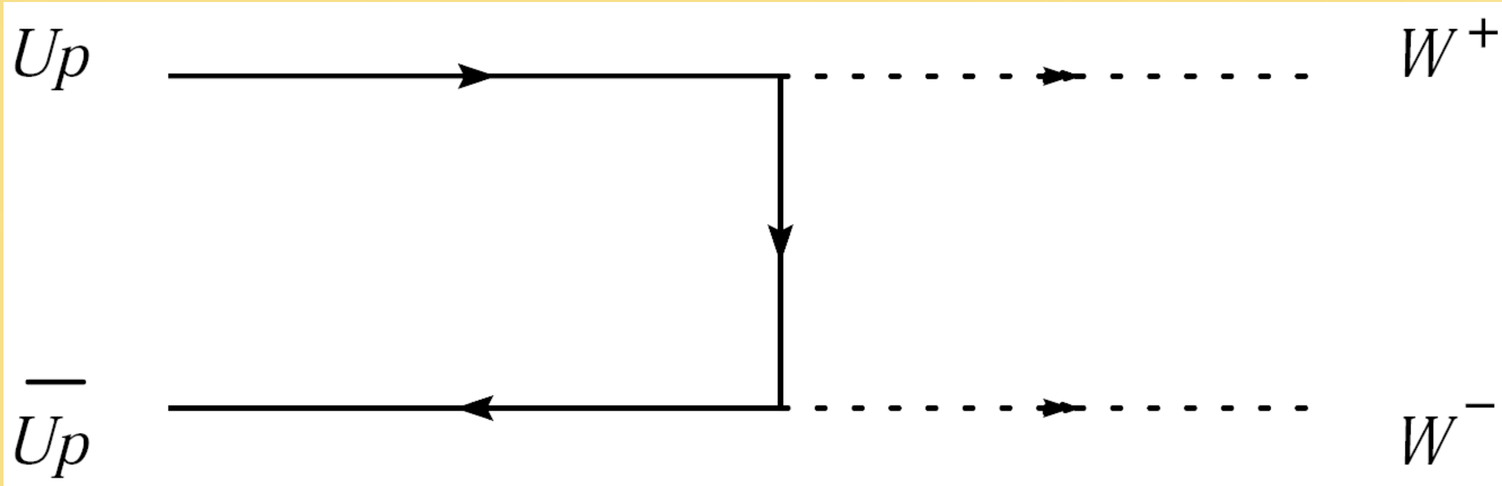
at NNLO:

$$d\sigma_n = d\sigma_n^{(\text{virtual diagrams})} + d\sigma_{n+1}^{(\text{virtual-real diagrams})}$$

$$+ \; d\sigma_{n+2}^{(\text{real diagrams})}$$

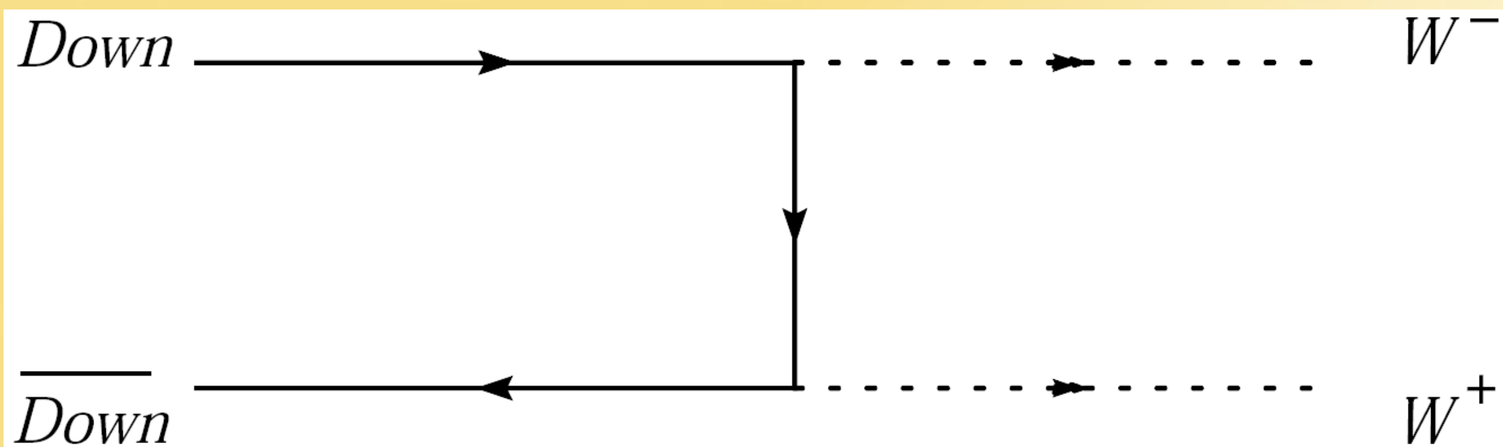
Difficulties lie at red (pink)

" Split the work in half… "



From an up-type diagram to a down-type diagram use the formal substitution

$W^+ \leftrightarrow W^-$



Amplitude decomposition:

The invariant squared amplitude for the Born process can be decomposed as :

$$\sum_{\text{spin,color}} |\mathcal{M}|^2 = N_c \,[c_i^{tt} F_i(s, t) - c_i^{ts} J_i(s, t) + c_i^{ss} K_i(s, t)]$$

where c_i^{tt}, c_i^{ts}, c_i^{ss} are “coupling constants”.

They generally depend on the mass M_Z, weak mixing angle, θ_w, charge and isospin of the quark

For the amplitude squared the change up-type $\leftrightarrow$ down-type is given by:

$$F_{down}(s, t) = F_{up}(s, u)$$

$$J_{down}(s, t) = -J_{up}(s, u)$$

$$K_{down}(s, t) = K_{up}(s, u)$$

and the corresponding changes to the couplings...

Technical details I

- A NNLO (4 legs, 2 loops) calculation of a process with massive particles (similar features to the recent "heavy quark production") Czakon, Mitov, Moch
- Color and spin averaged amplitudes (Here we present only the leading color coefficient)
- Kinematical region: all kinematical invariants large compared to the mass of W:

$$M_W^{\,2} \ll s, t, u$$

- Exact analytic result (up to terms suppressed by powers of $M_W^{\,2}$)

Technical details II

159 diagrams in total

after Reduction:

71 master integrals

35 needed for the leading color coefficient

real problem: Calculate the Masters!

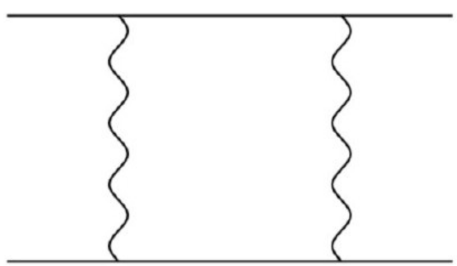
Way to go:

Mellin-Barnes representations

Mellin-Barnes representations
An example

$$\frac{1}{(A+B)^n} = \frac{1}{2\pi i \Gamma(n)} \int_{-i\infty}^{i\infty} dz \frac{A^z}{B^{n+z}} \Gamma(-z)\Gamma(n+z)$$

- Example



$$e^{\epsilon\gamma}\Gamma(2+\epsilon)\int dx_1 ... dx_4 \delta(1-x_1-...-x_4)\frac{1}{(-sx_2x_3-tx_1x_4)^{2+\epsilon}}$$

$$\frac{e^{\epsilon\gamma}}{2\pi i}\frac{1}{(-s)^{2+\epsilon}}\int_{-i\infty}^{i\infty} dz \ \left(\frac{t}{s}\right)^z \frac{\Gamma^2(-1-\epsilon-z)\Gamma(-z)\Gamma^2(1+z)\Gamma(2+\epsilon+z)}{\Gamma(-2\epsilon)}$$

$$\mathrm{Re}\ \epsilon = -\frac{1}{2}, \quad \mathrm{Re}\ z = -\frac{3}{4}$$

Software

MBrepresentations.m (GC, M. Czakon)

Produces representations for any multi-loop scalar and tensor integrals of any rank!

MB.m (M. Czakon)

Determination of contours, analytic continuation, expansion in a chosen parameter, numerical integration

XSummer (S. Moch, P. Uwer)

Evaluation of harmonic sums

PSLQ (D. Bailey)

Fitting to a transcendental basis

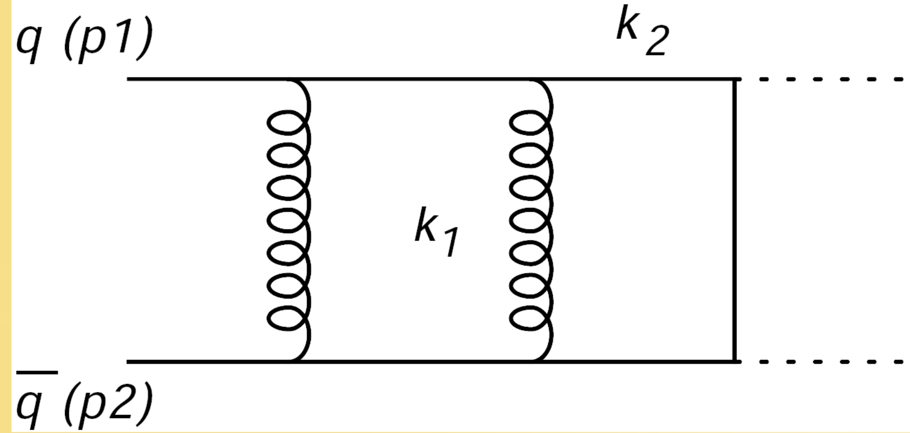
Outline of the Technique

Starting from the Feynman parameters representation of a diagram one obtains a multi-fold Mellin-Barnes integral representation.

The task then is to "walk" the following steps:

- produce representations (**MBrepresentations.m**)
- analytically continue in ε to the vivinity of 0 (**MB.m**)
- expand in mass (**MBasymptotics.m**, *M.Czakon*)
- perform as many as possible integrations using Barnes lemmas
- resum the remaining integrals by transforming into harmonic series (**Xsummer**)
- resum remaining constants by high-precision numerical evaluation and fitting it to a transcendental basis (**PSLQ**)

A tensor example:



(k1.p3) x

$$\int\int d^d k_1 d^d k_2 \frac{k_1.p_3}{k_1^2 k_2^2 (k_1+k_2)^2 (k_1+k_2-p_1)^2 (k_2-p_1-p_2)^2 (k_1+k_2-p_1-p_2)^2 (k_2-p_1-p_2+p_3)^2}$$

```
{(M2^(-1-ep-z5) S^(-2-ep-z1) U^(z1+z5) (-M2 + S + U) Gamma[-ep + z1] Gamma[2 + z2] Gamma[-1 - ep - z2 - z3]
   Gamma[-z3] Gamma[1 - z1 + z2 + z3] Gamma[-1 - ep - z1 + z3 - z4] Gamma[-z1 + z2 + z3 - z4]
   Gamma[-z4] Gamma[1 + z1 + z4] Gamma[2 + ep + z1 + z4] Gamma[1 - z3 + z4]
   Gamma[-z1 - z5] Gamma[1 - ep - z1 + z2 + z4 - z5] Gamma[1 + ep + z5] Gamma[z1 - z2 + z5]) /
 (2 Gamma[1 - 2 ep] Gamma[1 - z3] Gamma[1 - z1 + z2 + z3 - z4]
   Gamma[1 - 2 ep + z1 + z4] Gamma[2 - z1 + z2 + z4]),

 (M2^(-1-ep-z5) S^(-1-ep-z1) U^(z1+z5) Gamma[-ep + z1] Gamma[1 + z2] Gamma[-1 - ep - z2 - z3] Gamma[-z3]
   Gamma[1 - z1 + z2 + z3] Gamma[-ep - z1 + z3 - z4] Gamma[-z1 + z2 + z3 - z4] Gamma[-z4]
   Gamma[1 + z1 + z4] Gamma[2 + ep + z1 + z4] Gamma[1 - z3 + z4] Gamma[-z1 - z5]
   Gamma[1 - ep - z1 + z2 + z4 - z5] Gamma[1 + ep + z5] Gamma[z1 - z2 + z5]) /
 (2 Gamma[1 - 2 ep] Gamma[1 - z3] Gamma[1 - z1 + z2 + z3 - z4]
   Gamma[1 - 2 ep + z1 + z4] Gamma[2 - z1 + z2 + z4]),

 (M2^(-1-ep-z5) (2 M2 - S) S^(-2-ep-z1) U^(z1+z5) Gamma[1 - ep + z1] Gamma[2 + z2] Gamma[-1 - ep - z2 - z3]
   Gamma[-z3] Gamma[1 - z1 + z2 + z3] Gamma[-1 - ep - z1 + z3 - z4] Gamma[-z1 + z2 + z3 - z4]
   Gamma[-z4] Gamma[1 + z1 + z4] Gamma[2 + ep + z1 + z4] Gamma[1 - z3 + z4]
   Gamma[-z1 - z5] Gamma[1 - ep - z1 + z2 + z4 - z5] Gamma[1 + ep + z5] Gamma[z1 - z2 + z5]) /
 (2 Gamma[1 - 2 ep] Gamma[1 - z3] Gamma[1 - z1 + z2 + z3 - z4]
   Gamma[2 - 2 ep + z1 + z4] Gamma[2 - z1 + z2 + z4]), ...
```

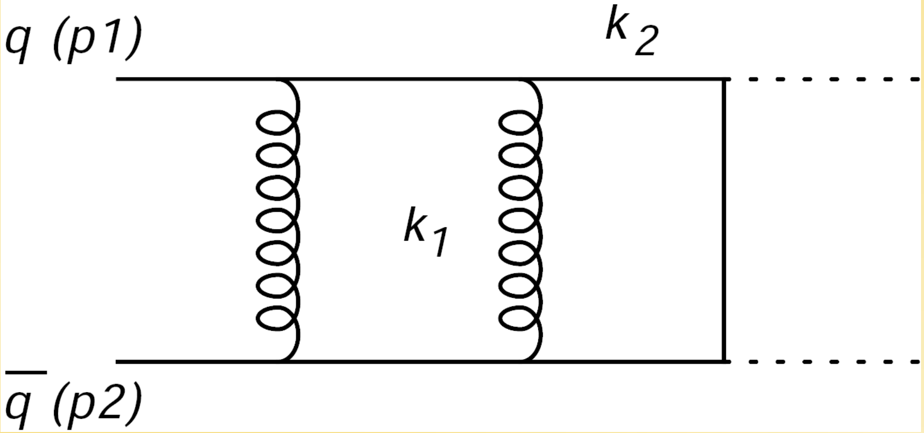
Mellin Barnes representation

11 terms in total

A tensor example:

$(k1.p3)\ x$

$(ms = M_W^{\,2}/s,\ x = -t/s)$

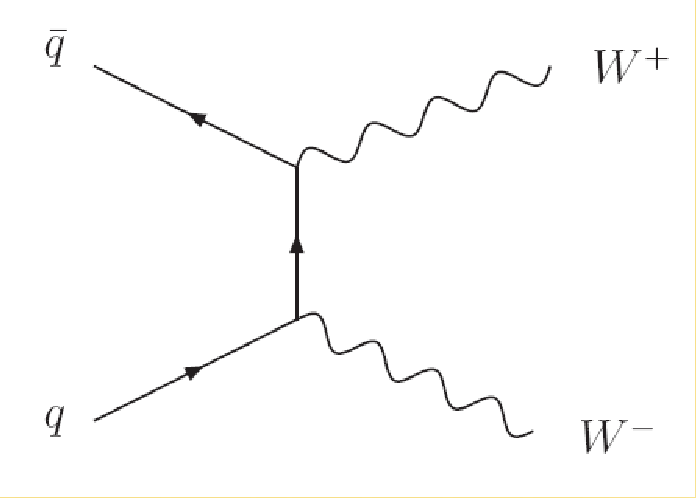


Analytic result

$$-\frac{1}{480\,(1-x)}$$

$$(40\,\pi^4 - 132\,\pi^4\,x + 360\,\pi^2\,H[0,1,x] - 80\,\pi^2\,x\,H[0,1,x] + 160\,\pi^2\,H[1,0,x] - 160\,\pi^2\,x\,H[1,0,x] +$$
$$620\,\pi^2\,H[1,1,x] - 400\,\pi^2\,x\,H[1,1,x] + 240\,H[0,0,1,1,x] + 480\,H[0,1,1,0,x] -$$
$$480\,x\,H[0,1,1,0,x] + 720\,H[0,1,1,1,x] + 480\,H[1,0,1,0,x] - 480\,x\,H[1,0,1,0,x] +$$
$$960\,H[1,0,1,1,x] + 480\,H[1,1,0,0,x] - 480\,x\,H[1,1,0,0,x] + 2400\,H[1,1,1,0,x] -$$
$$2400\,x\,H[1,1,1,0,x] + 1800\,H[1,1,1,1,x] + 100\,\pi^2\,H[1,x]\,\text{Log}[ms] -$$
$$240\,\pi^2\,x\,H[1,x]\,\text{Log}[ms] - 240\,H[0,1,1,x]\,\text{Log}[ms] + 1440\,H[1,1,0,x]\,\text{Log}[ms] -$$
$$1440\,x\,H[1,1,0,x]\,\text{Log}[ms] - 600\,H[1,1,1,x]\,\text{Log}[ms] + [illegible]0\,\pi^2\,\text{Log}[ms]^2 - 300\,\pi^2\,x\,\text{Log}[ms]^2 -$$
$$180\,H[1,1,x]\,\text{Log}[ms]^2 + 260\,H[1,x]\,\text{Log}[ms]^3 + 45\,\text{Log}[ms]^4 + 130\,x\,\text{Log}[ms]^4 +$$
$$160\,\pi^2\,H[0,x]\,\text{Log}[1-x] - 160\,\pi^2\,x\,H[0,x]\,\text{Log}[1-x] + 80\,\pi^2\,H[1,x]\,\text{Log}[1-x] -$$
$$80\,\pi^2\,x\,H[1,x]\,\text{Log}[1-x] + 480\,H[0,1,0,x]\,\text{Log}[1-x] - 480\,x\,H[0,1,0,x]\,\text{Log}[1-x] +$$
$$480\,H[1,0,0,x]\,\text{Log}[1-x] - 480\,x\,H[1,0,0,x]\,\text{Log}[1-x] + 480\,H[1,1,0,x]\,\text{Log}[1-x] -$$
$$480\,x\,H[1,1,0,x]\,\text{Log}[1-x] - 680\,\pi^2\,\text{Log}[ms]\,\text{Log}[1-x] + 680\,\pi^2\,x\,\text{Log}[ms]\,\text{Log}[1-x] +$$
$$1440\,H[1,0,x]\,\text{Log}[ms]\,\text{Log}[1-x] - 1440\,x\,H[1,0,x]\,\text{Log}[ms]\,\text{Log}[1-x] +$$
$$40\,\text{Log}[ms]^3\,\text{Log}[1-x] - 40\,x\,\text{Log}[ms]^3\,\text{Log}[1-x] + 300\,\pi^2\,\text{Log}[1-x]^2 -$$
$$300\,\pi^2\,x\,\text{Log}[1-x]^2 + 240\,H[0,0,x]\,\text{Log}[1-x]^2 - 240\,x\,H[0,0,x]\,\text{Log}[1-x]^2 -$$
$$720\,H[1,0,x]\,\text{Log}[1-x]^2 + 720\,x\,H[1,0,x]\,\text{Log}[1-x]^2 + 720\,H[0,x]\,\text{Log}[ms]\,\text{Log}[1-x]^2 -$$
$$720\,x\,H[0,x]\,\text{Log}[ms]\,\text{Log}[1-x]^2 - 60\,\text{Log}[ms]^2\,\text{Log}[1-x]^2 + 60\,x\,\text{Log}[ms]^2\,\text{Log}[1-x]^2 -$$
$$560\,H[0,x]\,\text{Log}[1-x]^3 + 560\,x\,H[0,x]\,\text{Log}[1-x]^3 - 200\,\text{Log}[ms]\,\text{Log}[1-x]^3 +$$
$$200\,x\,\text{Log}[ms]\,\text{Log}[1-x]^3 + 190\,\text{Log}[1-x]^4 - 190\,x\,\text{Log}[1-x]^4 -$$
$$6320\,H[1,x]\,\text{Zeta}[3] + 1920\,x\,H[1,x]\,\text{Zeta}[3] + 1280\,\text{Log}[ms]\,\text{Zeta}[3] -$$
$$1120\,x\,\text{Log}[ms]\,\text{Zeta}[3] - 2080\,\text{Log}[1-x]\,\text{Zeta}[3] + 2080\,x\,\text{Log}[1-x]\,\text{Zeta}[3])$$

Born result for the coupling c_i^{tt}



$$-\frac{4\,(-1+x)\,x}{ms^2}-\frac{8\,(-2+2\,ep+x)}{ms}-\frac{4\,(-1+(5-8\,ep+4\,ep^2)\,x)}{-1+x}$$

The amplitude in ms starts at order ms^{-2}

That is due to:

$$\sum_{\lambda=0,\pm1}\varepsilon_\mu(p,\lambda)\varepsilon_\nu(p,\lambda)=-g_{\mu\nu}+\frac{p_\mu p_\nu}{m^2}$$

"Singular behavior of QCD amplitudes", recipe by Catani:

One loop: The IR pole structure of the renormalized amplitude can be known by only knowing the tree level amplitude:

$$|\mathcal{M}_m^{(1)}(\mu^2;\{p\})\rangle_{\rm R.S.} = \boldsymbol{I}^{(1)}(\epsilon,\mu^2;\{p\})\ |\mathcal{M}_m^{(0)}(\mu^2;\{p\})\rangle_{\rm R.S.} + |\mathcal{M}_m^{(1)\,{\rm fin}}(\mu^2;\{p\})\rangle_{\rm R.S.}$$

Two loop: Now you need tree and one loop level amplitude:

$$\begin{aligned}|\mathcal{M}_m^{(2)}(\mu^2;\{p\})\rangle_{\rm R.S.} &= \boldsymbol{I}^{(1)}(\epsilon,\mu^2;\{p\})\ |\mathcal{M}_m^{(1)}(\mu^2;\{p\})\rangle_{\rm R.S.} \\ &+ \boldsymbol{I}_{\rm R.S.}^{(2)}(\epsilon,\mu^2;\{p\})\ |\mathcal{M}_m^{(0)}(\mu^2;\{p\})\rangle_{\rm R.S.} + |\mathcal{M}_m^{(2){\rm fin}}(\mu^2;\{p\})\rangle_{\rm R.S.}\end{aligned}$$

Singular dependence embodied in the operators $I^{(1)}$ and $I^{(2)}$

Leading color coefficient for the coupling c_i^{tt}

```
-((-1 + x)^2 x + ms^2 (-1 + 5 x) + 2 ms (2 - 3 x + x^2))/(8 ms^2 (-1 + x) ε^4) +
1/(16 ms^2 (-1 + x) ε^3) (ms^2 (17 - 69 x) - 17 (-1 + x)^2 x + ms (-60 + 94 x - 34 x^2) +
  4 ((-1 + x)^2 x + ms^2 (-1 + 5 x) + 2 ms (2 - 3 x + x^2)) Log[s]) + 1/(288 ms^2 (-1 + x) ε^2)
(-508 ms + 433 ms^2 + 288 ms π^2 - 72 ms^2 π^2 - 433 x + 1374 ms x - 293 ms^2 x +
  72 π^2 x - 432 ms π^2 x + 360 ms^2 π^2 x + 866 x^2 - 866 ms x^2 + 288 ms^2 x^2 -
  144 π^2 x^2 + 144 ms π^2 x^2 - 433 x^3 + 72 π^2 x^3 + 288 ms^2 (-1 + x + x^2) Log[ms] +
  12 (29 (-1 + x)^2 x + ms^2 (-29 + 97 x) + 2 ms (46 - 75 x + 29 x^2)) Log[s] -
  72 ((-1 + x)^2 x + ms^2 (-1 + 5 x) + 2 ms (2 - 3 x + x^2)) Log[s]^2 +
  144 ms^2 x Log[1 - x] + 144 ms^2 x Log[1 - x]^2) - 1/(1728 ms^2 (-1 + x) ε)
(5788 ms - 4045 ms^2 - 4248 ms π^2 + 1494 ms^2 π^2 + 4045 x - 13878 ms x + 14129 ms^2 x - 1494 π^2 x +
  7236 ms π^2 x - 2718 ms^2 π^2 x - 8090 x^2 + 8090 ms x^2 - 6048 ms^2 x^2 + 2988 π^2 x^2 - 2988 ms π^2 x^2 +
  4045 x^3 - 1494 π^2 x^3 + 864 ms^2 (-1 + x + x^2) Log[ms]^2 - 2928 ms Log[s] + 2820 ms^2 Log[s] +
  3456 ms π^2 Log[s] - 864 ms^2 π^2 Log[s] - 2820 x Log[s] + 8568 ms x Log[s] - 4308 ms^2 x Log[s] +
  864 π^2 x Log[s] - 5184 ms π^2 x Log[s] + 4320 ms^2 π^2 x Log[s] + 5640 x^2 Log[s] -
  5640 ms x^2 Log[s] + 3456 ms^2 x^2 Log[s] - 1728 π^2 x^2 Log[s] + 1728 ms π^2 x^2 Log[s] -
  2820 x^3 Log[s] + 864 π^2 x^3 Log[s] + 3456 ms Log[s]^2 - 1296 ms^2 Log[s]^2 + 2[illegible] x Log[s]^2 -
  6048 ms x Log[s]^2 + 3024 ms^2 x Log[s]^2 - 259[illegible] x^2 Log[s]^2 + [illegible]592 ms x^2 Log[s]^2 +
  1296 x^3 Log[s]^2 - 1152 ms Log[s]^3 + [illegible]8[illegible] ms^2 Log[s]^3 - [illegible] x Log[s]^3 + 1728 ms x Log[s]^3 -
  1440 ms^2 x Log[s]^3 + 576 x^2 Log[s]^3 - 576 ms x^2 Log[s]^3 - 288 x^3 Log[s]^3 +
  864 ms^2 Log[ms] (7 - 9 [illegible] x^2 + 4 (-1 + x + x^2) Log[s]) + 1728 ms^2 Log[1 - x] -
  2160 ms^2 x Log[1 - x] + 25[illegible]2 ms^2 π^2 x Log[1 - x] + 1728 ms^2 x Log[s] Log[1 - x] +
  864 ms^2 Log[1 - x]^2 + 1728 ms^2 x Log[s] Log[1 - x]^2 + 576 ms^2 x Log[1 - x]^3 -
  1728 ms^2 x S[1, 2, x] + 2016 ms Zeta[3] - 504 ms^2 Zeta[3] + 504 x Zeta[3] - 3024 ms x Zeta[3] +
  2520 ms^2 x Zeta[3] - 1008 x^2 Zeta[3] + 1008 ms x^2 Zeta[3] + 504 x^3 Zeta[3]) +
1/(51840 ms^2 (-1 + x) x^2) (25920 ms^2 π^2 x + 667060 ms x^2 - 45415 ms^2 x^2 + 79080 ms π^2 x^2 -
  48750 ms^2 π^2 x^2 - 18936 ms π^4 x^2 + 11070 ms^2 π^4 x^2 + 45415 x^3 - 757890 ms x^3 +
  1082675 ms^2 x^3 + 64590 π^2 x^3 - 208260 ms π^2 x^3 + 185430 ms^2 π^2 x^3 - 4734 π^4 x^3 +
  28404 ms π^4 x^3 - 29142 ms^2 π^4 x^3 - 90830 x^4 + 90830 ms x^4 - 36000 ms^2 x^4 - 129180 π^2 x^4 +
  129180 ms π^2 x^4 - 93600 ms^2 π^2 x^4 + 9468 π^4 x^4 - 9468 ms π^4 x^4 + 45415 x^5 + 64590 π^2 x^5 -
  4734 π^4 x^5 + 165600 ms^2 x^2 Log[ms] + 112320 ms^2 π^2 x^2 Log[ms] - 61920 ms^2 x^3 Log[ms] -
  112320 ms^2 π^2 x^3 Log[ms] - 53280 ms^2 x^4 Log[ms] - 112320 ms^2 π^2 x^4 Log[ms] +
  90720 ms^2 x^2 Log[ms]^2 - 129600 ms^2 x^3 Log[ms]^2 - 64800 ms^2 x^4 Log[ms]^2 -
  8640 ms^2 x^2 Log[ms]^3 + 8640 ms^2 x^3 Log[ms]^3 + 8640 ms^2 x^4 Log[ms]^3 - 127920 ms x^2 Log[s] -
  52620 ms^2 x^2 Log[s] - 144000 ms π^2 x^2 Log[s] + 61920 ms^2 π^2 x^2 Log[s] +
  52620 x^3 Log[s] + 22680 ms x^3 Log[s] - 7620 ms^2 x^3 Log[s] - 61920 π^2 x^3 Log[s] +
  267840 ms π^2 x^3 Log[s] - 24480 ms^2 π^2 x^3 Log[s] - 105240 x^4 Log[s] + 105240 ms x^4 Log[s] -
  172800 ms^2 x^4 Log[s] + 123840 π^2 x^4 Log[s] - 123840 ms π^2 x^4 Log[s] + 52620 x^5 Log[s] -
  61920 π^2 x^5 Log[s] + 172800 ms^2 x^2 Log[ms] Log[s] - 276480 ms^2 x^3 Log[ms] Log[s] -
  69120 ms^2 x^4 Log[ms] Log[s] - 51840 ms^2 x^2 Log[ms]^2 Log[s] + 51840 ms^2 x^3 Log[ms]^2 Log[s] +
  51840 ms^2 x^4 Log[ms]^2 Log[s] - 40320 ms x^2 Log[s]^2 + 48960 ms^2 x^2 Log[s]^2 +
  103680 ms π^2 x^2 Log[s]^2 - 25920 ms^2 π^2 x^2 Log[s]^2 - 48960 x^3 Log[s]^2 +
  138240 ms x^3 Log[s]^2 - 141120 ms^2 x^3 Log[s]^2 + 25920 π^2 x^3 Log[s]^2 -
  155520 ms π^2 x^3 Log[s]^2 + 129600 ms^2 π^2 x^3 Log[s]^2 + 97920 x^4 Log[s]^2 -
  97920 ms x^4 Log[s]^2 + 103680 ms^2 x^4 Log[s]^2 - 51840 π^2 x^4 Log[s]^2 + 51840 ms π^2 x^4 Log[s]^2 -
  48960 x^5 Log[s]^2 + 25920 π^2 x^5 Log[s]^2 - 103680 ms^2 x^2 Log[ms] Log[s]^2 +
  103680 ms^2 x^3 Log[ms] Log[s]^2 + 103680 ms^2 x^4 Log[ms] Log[s]^2 + 3744[illegible] x[illegible] Log[s]^3 -
  18000 ms^2 x^2 Log[s]^3 + 18000 x^3 Log[s]^3 - 73440 m[illegible] x^3 Log[s]^3 + 208[illegible]0 [illegible]s^2 x^3 Log[s]^3 -
  36000 x^4 Log[s]^3 + 3600[illegible] ms x^4 Log[s]^3 [illegible] 80[illegible]0 x^5 Log[s]^3 - 172[illegible]0 ms x^2 Log[s]^4 +
  4320 ms^2 x^2 Log[s]^4 [illegible] 43[illegible]0 x^3 Log[s]^4 + 2[illegible]920 ms x[illegible] Log[s]^4 - 21600 ms^2 x^3 Log[s]^4 +
  864[illegible] x^4 Log[s]^4 [illegible] 8[illegible]40 ms x^4 Log[s]^4 - 4320 x^5 Log[s]^4 + 25920 ms^2 π^2 Log[1 - x] +
  51[illegible]40 ms^2 x Log[1 - x] - 311040 [illegible]s^2 x^2 Log[1 - x] + 95040 ms^2 π^2 x^2 Log[1 - x] -
  [illegible] ms^2 x^3 Log[1 - x] + 24480 ms^2 π^2 x^3 Log[1 - x] + 164160 ms^2 x^4 Log[1 - x] -
  8640 ms^2 π^2 x^4 Log[1 - x] + 51840 ms^2 x Log[ms] Log[1 - x] - 164160 ms^2 x^2 Log[ms] Log[1 - x] +
  138240 ms^2 x^3 Log[ms] Log[1 - x] + 112320 ms^2 x^4 Log[ms] Log[1 - x] +
  103680 ms^2 x^2 Log[s] Log[1 - x] - 34560 ms^2 x^3 Log[s] Log[1 - x] +
  155520 ms^2 π^2 x^3 Log[s] Log[1 - x] + 51840 ms^2 x^3 Log[s]^2 Log[1 - x] + 51840 ms^2 Log[1 - x]^2 -
  155520 ms^2 x Log[1 - x]^2 + 181440 ms^2 x^2 Log[1 - x]^2 - 227520 ms^2 x^3 Log[1 - x]^2 +
  56160 ms^2 π^2 x^3 Log[1 - x]^2 - 25920 ms^2 x^4 Log[1 - x]^2 + 25920 ms^2 Log[ms] Log[1 - x]^2 -
  25920 ms^2 x^3 Log[ms] Log[1 - x]^2 - 25920 ms^2 x^4 Log[ms] Log[1 - x]^2 +
  51840 ms^2 x^2 Log[s] Log[1 - x]^2 + 95040 ms^2 x^3 Log[s] Log[1 - x]^2 +
  51840 ms^2 x^3 Log[s]^2 Log[1 - x]^2 - 8640 ms^2 Log[1 - x]^3 + 63360 ms^2 x^3 Log[1 - x]^3 +
  8640 ms^2 x^4 Log[1 - x]^3 + 34560 ms^2 x^3 Log[s] Log[1 - x]^3 - 69120 ms^2 x^3 Log[1 - x]^3 Log[x] -
  103680 ms^2 x^3 Log[1 - x]^2 PolyLog[2, 1 - x] - 34560 ms^2 π^2 x^2 PolyLog[2, x] +
  95040 ms^2 π^2 x^3 PolyLog[2, x] + 207360 ms^2 x^3 PolyLog[4, 1 - x] -
  51840 ms^2 (1 - x^2 + 2 x^3 + x^4 + 2 x^3 Log[s] - 2 x^3 Log[1 - x]) S[1, 2, x] +
  51840 ms^2 x^2 (-4 + 3 x) S[2, 2, x] - 89280 ms x^2 Zeta[3] + 37440 ms^2 x^2 Zeta[3] -
  37440 x^3 Zeta[3] + 164160 ms x^3 Zeta[3] + 11520 ms^2 x^3 Zeta[3] + 74880 x^4 Zeta[3] -
  74880 ms x^4 Zeta[3] - 37440 x^5 Zeta[3] + 120960 ms x^2 Log[s] Zeta[3] -
  30240 ms^2 x^2 Log[s] Zeta[3] + 30240 x^3 Log[s] Zeta[3] - 181440 ms x^3 Log[s] Zeta[3] +
  151200 ms^2 x^3 Log[s] Zeta[3] - 60480 x^4 Log[s] Zeta[3] + 60480 ms x^4 Log[s] Zeta[3] +
  30240 x^5 Log[s] Zeta[3] - 51840 ms^2 x^3 Log[1 - x] Zeta[3])
```

in agreement with catani prediction

$ms = M_W^2/s, \quad x = -t/s$

Conclusions

- Mellin Barnes representations approach is a powerful technique
- Not easy though (especially for the non-planar graphs)
- We have finally the full result
- Next to come are higher power corrections

Outlook

- Next process at NNLO :

$$gg \rightarrow W^+W^-$$

- A NNLO Monte Carlo generator
  Real corrections needed, a possible treatment is with sectors decomposition