Radiative Corrections in High-Energy Physics

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The LHC

The LHC gives us the opportunity to look at particle processes with

- energy scales $\approx 1 - 2\,\mathrm{TeV}$ with high statistics
- maximum of energy $\approx 14\,\mathrm{TeV}$

We would like to find

- the Higgs boson
- evidences for new physics e.g.
  - SUSY particles $\rightarrow$ Supersymmetry
  - anomalous triple/quartic gauge-boson couplings $\rightarrow \mathrm{SU}(2) \times \mathrm{U}(1)$?
  - Kaluza-Klein states $\rightarrow$ Extra dimensions

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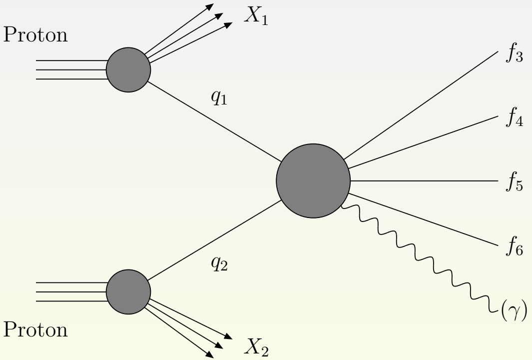
- a lot of particles we already know
- more "Higgses" and of course at least one of these guys!

Processes at a hadron machine

The Parton Model:



The cross section is calculated assuming a hard scattering process of the partons q_1 and q_2.

Collisions of protons

The parton model in a formula:

Cross sections for $\mathrm{pp} \longrightarrow 4f(+\gamma)$:

$$\mathrm{d}\sigma_{q_1 q_2} = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \, \Phi_{q_1}(x_1, Q) \, \Phi_{q_2}(x_2, Q) \, \mathrm{d}\hat{\sigma}_{q_1 q_2}(x_1, x_2)$$

where

- $\Phi_q(x, Q)$ - parton density functions
- $\hat{\sigma}_{q_1 q_2}$ - differential cross section of the underlying hard scattering process

Inclusion of radiative corrections

Corrections from different parts of the theory

- QCD $\rightarrow$ corrections in $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$
- QED $\rightarrow$ corrections in $\mathcal{O}(\alpha)$
- EW $\rightarrow$ corrections in $\mathcal{O}(\frac{\alpha}{s_\mathrm{w}^2})$

Hierarchy of the different corrections:

- QCD $\sim \alpha_s \approx 0.1$ or $\sim \alpha_s^2 \approx 0.01$
- QED $\sim \alpha \approx 0.01$
- EW $\sim \alpha/s_\mathrm{w}^2 \approx 0.035$

The most important corrections to hadronic processes arise from QCD
(see e.g. Dixon, Kunszt, Signer in Phys.Rev.D60:114037,1999)

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Properties of different corrections

In QCD and QED we have to sum the real and virtual corrections!
$\Rightarrow$ Cancellations of IR singularities between virtual and real corrections

In the electroweak theory we have massive gauge bosons with $M_{\mathrm{Z}} \approx 91$ GeV and $M_{\mathrm{W}} \approx 80$ GeV.
$\Rightarrow$ No 'soft' or 'collinear' divergences due to massive gauge bosons!
$\Rightarrow$ Emission of real Z or W can be experimentally separated
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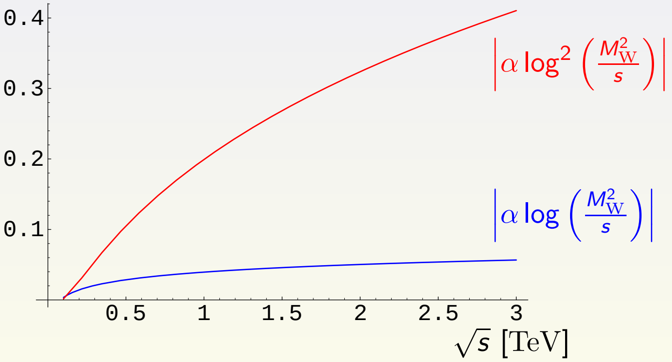
⇒ No 'soft' or 'collinear' divergences due to massive gauge bosons!
⇒ Emission of real Z or W can be experimentally separated
⇒ Mass singular virtual corrections due to Z and W are not cancelled by the corresponding real corrections
⇒ Large logarithms at high energy scales:
EW correction terms $\sim \alpha \log\left(\frac{M_{\mathrm{W}}^2}{s}\right)$ and $\sim \alpha \log^2\left(\frac{M_{\mathrm{W}}^2}{s}\right)$

EW Logarithms



Electroweak correction terms may get large for $\sqrt{s} > 0.5$ TeV!

Leading logarithmic corrections

In the high energy processes we find

- intermediate masses M_V of the massive gauge bosons
- large kinematic invariants $s = (p_i + p_j)^2$ and $t = (p_i - p_j)^2$
- small masses m_f of the external fermions

Definition of leading logarithmic corrections

We use

- that all invariants $s, t \gg M_\mathrm{Z}^2 \approx M_\mathrm{W}^2 \gg m_f^2 \gg \lambda^2$
- $m_f \to 0$ for all fermions exept of the top quark
- in dimensional regularization $\mu^2 = s = (p_1 + p_2)^2$

$\Rightarrow$ We only keep terms including $\alpha \log^2 \frac{s}{M_\mathrm{W}^2}$ and $\alpha \log \frac{s}{M_\mathrm{W}^2}$

Leading logarithmic corrections

The leading logarithmic corrections have been investigated for a long time by many different working groups

- Kühn et al.
- Ciafaloni, Comelli
- Fadin, Khoze, Melles
- Beenakker, Werthenbach
- Denner, Pozzorini

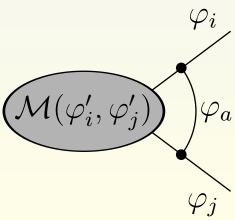
Leading logarithmic corrections

Approximation of loop integrals / eikonal approximation

$$J = -\mathrm{i}(4\pi)^2\mu^{4-D} \times \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{N(q)}{D_0 D_1 D_2}$$

with $q \to 0$ and $M_\varphi \to 0$ in the numerator $N(q)$

$$D_0 = q^2 - M_a^2 + \mathrm{i}\varepsilon$$
$$D_1 = (q - p_i)^2 - M_{\varphi_i'}^2 + \mathrm{i}\varepsilon$$
$$D_2 = (q + p_j)^2 - M_{\varphi_j'}^2 + \mathrm{i}\varepsilon$$



Leading logarithmic corrections

Approximation of loop integrals / eikonal approximation

$$J_{\mathrm{eik}} = \frac{N_{\mathrm{eik}}(0)}{(p_i+p_j)^2}\left\{\frac{1}{2}\log^2\left(\frac{-(p_i+p_j)^2-\mathrm{i}\varepsilon}{M_{\varphi_a}^2-\mathrm{i}\varepsilon}\right) + I_C(p_i^2, M_{\varphi_a}, M_{\varphi'_i}) + I_C(p_j^2, M_{\varphi_a}, M_{\varphi'_j})\right\}$$

with

$$I_C(p^2, M_0, M_1) = -\int_0^1 \frac{\mathrm{d}x}{x}\log\left(1+\frac{M_1^2-M_0^2-p^2}{M_0^2-\mathrm{i}\varepsilon}x+\frac{p^2}{M_0^2-\mathrm{i}\varepsilon}x^2\right)$$

Denner, Roth

Generic form of the corrections

The virtual leading logarithmic corrections have the general form:

$$\delta\mathcal{M}^{\varphi_1\cdots\varphi_n}(\{C_i\},p_1,\ldots,p_n) = \sum_{\varphi'_1\cdots\varphi'_n} \delta^{\varphi_1\cdots\varphi_n}_{\varphi'_1\cdots\varphi'_n}\,\mathcal{M}_0^{\varphi'_1\cdots\varphi'_n}(\{C_i\},p_1,\ldots,p_n) + \sum_{C_i} \underbrace{\delta C_i\,\frac{\partial\mathcal{M}_0^{\varphi_1\cdots\varphi_n}}{\partial C_i}(\{C_i\},p_1,\ldots,p_n)}_{\text{Parameter Renormalization }\ \delta^{\mathrm{PR}}}$$

For $\delta^{\varphi_1\cdots\varphi_n}_{\varphi'_1\cdots\varphi'_n}$ we distinguish the terms

- δ^{LSC} Leading Soft Collinear mass singularities
- δ^{SSC} Subleading Soft Collinear contributions
- δ^{C} Collinear logarithms and corrections to the wave functions

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Formulas for the leading electroweak radiative corrections

The other correction factors have similar forms!

- $\delta^{\mathrm{LSC}} \propto \log^2$
  - requires $\mathcal{M}_0$ with 1 replaced particle $(A \leftrightarrow Z)$
- $\delta^{\mathrm{SSC}} \propto \log$
  - angular dependent
  - requires $\mathcal{M}_0$ with 2 replaced particles $(A \leftrightarrow Z), (W^{\pm} \leftrightarrow A, Z), (Z^{L} \leftrightarrow H), (u \leftrightarrow d), \mathrm{etc.}$
- $\delta^{\mathrm{C}} \propto \log$
  - include mixing of A and Z
- $\delta^{\mathrm{PR}} \propto \log$
  - from renormalization of e and c_{w}
  - requires exchange of couplings by counterterm couplings

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- $\delta^{\mathrm{C}} \propto \log$ with $\log(s/M_{\mathrm{W}}^2)$ and $\log(M_{\mathrm{W}}^2/\lambda^2)$ or $\log(M_{\mathrm{W}}^2/m_f^2)$
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- $\delta^{\mathrm{PR}} \propto \log$ with $\log(s/M_{\mathrm{W}}^2)$ and $\log(M_{\mathrm{W}}^2/\lambda^2)$ or $\log(M_{\mathrm{W}}^2/m_f^2)$
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Formulas for the leading electroweak radiative corrections

The generic form of these corrections has been calculated by Ansgar Denner and Stefano Pozzorini

Example of the leading soft-collinear corrections:

For an arbitrary process involving n external particles $\varphi_1, \ldots, \varphi_n$ we find:

$$\delta^{\mathrm{LSC}} \mathcal{M}^{\varphi_1 \ldots \varphi_n} = \sum_{k=1}^{n} \sum_{\varphi'_k} \delta^{\mathrm{LSC}}_{\varphi'_k \varphi_k} \mathcal{M}_0^{\varphi_1 \ldots \varphi'_k \ldots \varphi_n}$$

with $\delta^{\mathrm{LSC}}_{\varphi'_k \varphi_k} \propto \delta_{\varphi'_k \varphi_k}$ for all particles except of the γ and the Z

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Formulas for the leading electroweak radiative corrections

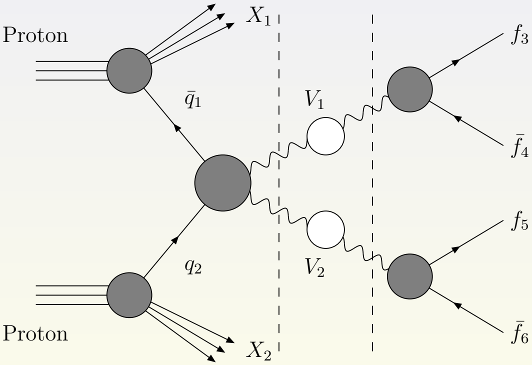
Explicit form of the correction factor:

$$\delta^{\mathrm{LSC}}_{\varphi'_k\varphi_k} = -\frac{\alpha}{8\pi} C^{\mathrm{ew}}_{\varphi'_k\varphi_k} \log^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) + \delta_{\varphi'_k\varphi_k}\left\{\frac{\alpha}{4\pi}(I^Z)^2_{\varphi_k} \log\left(\frac{s}{M_{\mathrm{W}}^2}\right)\log\left(\frac{M_{\mathrm{Z}}^2}{M_{\mathrm{W}}^2}\right) - \frac{1}{2} Q^2_{\varphi_k} L^{\mathrm{em}}(s,\lambda^2,M^2_{\varphi_k})\right\}$$

with the electromagnetic logarithms given by

$$L^{\mathrm{em}}(s,\lambda^2,M^2_{\varphi}) := \frac{\alpha}{4\pi}\left\{2\log\left(\frac{s}{M_{\mathrm{W}}^2}\right)\log\left(\frac{M_{\mathrm{W}}^2}{\lambda^2}\right) + \log^2\left(\frac{M_{\mathrm{W}}^2}{\lambda^2}\right) - \log^2\left(\frac{M_{\varphi}^2}{\lambda^2}\right)\right\}$$

Gauge-Boson pair production



The signature for gauge-boson pair production processes

- in total 4 fermions in the final state
- two fermion pairs which form massive gauge bosons

Our approach

We want to achieve an overall precision of $\approx 5\%$ for our prediction

We have to calculate

- a complicated phase-space integral (8 and 11 dimensions)
- a convolution over structure functions (2 more dimensions)
- the virtual QED and EW radiative corrections
- the real QED corrections
- the $\overline{MS}$ renormalisation of the PDFs

Our solution:

- A multi-channel Monte Carlo program for the integration
- the use of high-energy approximation ($\Delta \sim \mathcal{O}(\frac{\alpha}{\pi}\mathrm{const}) \lesssim 5\%$)
- and double pole approximations ($\Delta \sim \mathcal{O}(\frac{\alpha}{\pi}\frac{\Gamma_V}{M_V}) \lesssim 1\%$)

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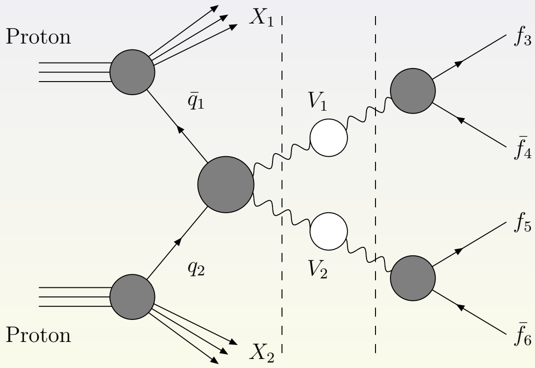
Double pole approximation

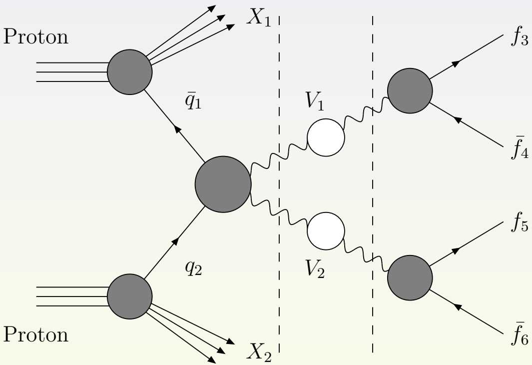


In DPA one divides the process into

- gauge-boson production
- gauge-boson decay

Aeppli, v. Oldenborgh, Wyler
Beenakker, Berends, Chapovsky
Denner, Dittmaier, Roth, Wackeroth

Double pole approximation



We have to calculate radiative corrections to

- gauge-boson production
- gauge-boson decay
- the non-factorizable parts

AEPPLI, V. OLDENBORGH, WYLER
BEENAKKER, BERENDS, CHAPOVSKY
DENNER, DITTMAIER, ROTH, WACKEROTH

The virtual corrections

$$\delta\mathcal{M}^{\mathrm{NLO}}_{\mathrm{virt}} = \delta\mathcal{M}^{\mathrm{DPA}}_{\mathrm{production}} + \delta\mathcal{M}^{\mathrm{DPA}}_{\mathrm{decay}} + \delta\mathcal{M}^{\mathrm{DPA}}_{\mathrm{non-fac}}$$

with the production subcontribution given by

$$\delta\mathcal{M}^{\mathrm{DPA}}_{\mathrm{production}} = \sum_{\tau_1,\tau_2} \delta\mathcal{M}_{f_1 f_2 \to V_1^{\tau_1} V_2^{\tau_2}} P_{V_1} P_{V_2} \times \mathcal{M}_{V_1^{\tau_1} \to f_3 \bar{f}_4} \mathcal{M}_{V_2^{\tau_2} \to f_5 \bar{f}_6}$$

where

$$\delta\mathcal{M}_{f_1 f_2 \to V_1^{\tau_1} V_2^{\tau_2}} = \delta^{\mathrm{LSC}}\mathcal{M}_{f_1 f_2 \to V_1^{\tau_1} V_2^{\tau_2}} + \delta^{\mathrm{SSC}}\mathcal{M}_{f_1 f_2 \to V_1^{\tau_1} V_2^{\tau_2}} + \delta^{\mathrm{C}}\mathcal{M}_{f_1 f_2 \to V_1^{\tau_1} V_2^{\tau_2}} + \delta^{\mathrm{PR}}\mathcal{M}_{f_1 f_2 \to V_1^{\tau_1} V_2^{\tau_2}}$$

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Virtual corrections

The decay subcontributions

$$\delta\mathcal{M}^{\mathrm{DPA}}_{\mathrm{decay}} = \sum_{\tau_1,\tau_2} \mathcal{M}_{f_1 f_2 \to V_1^{\tau_1} V_2^{\tau_2}} P_{V_1} P_{V_2} \times \left(\delta\mathcal{M}_{V_1^{\tau_1} \to f_3 \bar{f}_4} \mathcal{M}_{V_2^{\tau_2} \to f_5 \bar{f}_6} + \mathcal{M}_{V_1^{\tau_1} \to f_3 \bar{f}_4} \delta\mathcal{M}_{V_2^{\tau_2} \to f_5 \bar{f}_6} \right)$$

with

$$\delta\mathcal{M}_{V^\tau \to f_i \bar{f}_j} = \delta_{V \to f_i \bar{f}_j} \mathcal{M}_{V^\tau \to f_i \bar{f}_j}$$

and simple correction factors $\delta_{V \to f_i \bar{f}_j}$ for Z and W bosons.

Virtual corrections

The non-factorizable subcontribution is

$$\delta\mathcal{M}^{\mathrm{DPA}}_{\mathrm{non-fac}} = \frac{1}{2}\delta^{\mathrm{non-fac}}\mathcal{M}^{\mathrm{DPA}}_{\mathrm{Born}}$$

BEENAKKER, BERENDS, CHAPOVSKY
DENNER, DITTMAIER, ROTH

with

$$\begin{aligned}\delta^{\mathrm{non-fac}} &= \frac{\alpha}{\pi}\Bigg(- \sum_{i=3}^{4}\sum_{j=5}^{6} Q_i Q_j \theta_d(i)\theta_d(j)\mathrm{Re}\{\Delta_1(k_1, p_i, k_2, p_j)\} \\ &+ \sum_{n=1}^{2}\sum_{i=3}^{4} Q_n Q_i \theta_d(n)\theta_d(i)\mathrm{Re}\{\Delta_2(p_n, k_1, p_i)\} \\ &+ \sum_{n=1}^{2}\sum_{j=5}^{6} Q_n Q_j \theta_d(n)\theta_d(j)\mathrm{Re}\{\Delta_2(p_n, k_2, p_j)\}\Bigg)\end{aligned}$$

and two functions $\Delta_1(k_1, p_i, k_2, p_j)$ and $\Delta_2(p_n, k_1, p_i)$.

Real corrections / Matching of infrared divergences

The real corrections consist of the cross section for the emission of one additional photon.

- The full process for $\mathrm{pp} \to 4f + \gamma$ was calculated
- The infrared divergent part of the virtual corrections was subtracted using DPA:

$$\sigma_{\mathrm{virt,finite}} = \int_{\Phi_4^{\mathrm{DPA}}} \left(\mathrm{d}\sigma_{\mathrm{virt}} - \delta_{\mathrm{IR}}\, \mathrm{d}\sigma_{\mathrm{Born}}^{\mathrm{DPA}} \right)$$

- It was added to the real cross section using the full Born

$$\sigma'_{\mathrm{real}} = \int_{\Phi_5} \mathrm{d}\sigma_{\mathrm{real}} + \int_{\Phi_4} \delta_{\mathrm{IR}}\, \mathrm{d}\sigma_{\mathrm{Born}}$$

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$$\sigma_{\text{virt,finite}} = \int_{\Phi_4^{\text{DPA}}} \left(\mathrm{d}\sigma_{\text{virt}} - \delta_{\text{IR}}\, \mathrm{d}\sigma_{\text{Born}}^{\text{DPA}}\right)$$

- It was added to the real cross section using the full Born

$$\sigma'_{\text{real}} = \int_{\Phi_5} \mathrm{d}\sigma_{\text{real}} + \int_{\Phi_4} \delta_{\text{IR}}\, \mathrm{d}\sigma_{\text{Born}}$$

$\Rightarrow$ The infrared divergences cancel !

Double pole / High energy approximation

- Advantages of the DPA / High energy approximation:
  - Reduction of the number of Feynman diagrams
  - Superior to a simple *production* $\times$ *decay* approach
  - All loop integrals can be calculated in this approximation
  - State of the art for many-particle final states
  - Generic corrections for all final states
- Disadvantages of the DPA / High energy approximation
  - Missing 'non-logarithmic' terms in the radiative corrections
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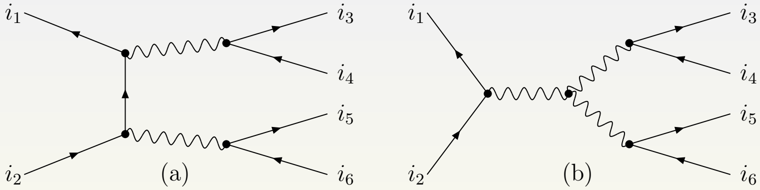
Construction of matrix elements

Two fundamental topologies exist:



$\mathcal{M}^{\mathrm{a},V_1,V_2}(i_1,i_2,i_3,i_4,i_5,i_6)$ $\mathcal{M}^{\mathrm{b},V_3}(i_1,i_2,i_3,i_4,i_5,i_6)$

All Feynman diagrams can be written as a permutation of the external legs in the topologies (a) or (b).

RacoonWW
Denner, Dittmaier, Roth, Wackeroth

Construction of matrix elements

The case of $N_{\rm quarks} \leq 2$ is particularly easy:

$$|\mathcal{M}_{\rm Born}|^2 = \frac{1}{4} \frac{N_{\rm colour}}{N_{\rm av} N_{\rm perm}} \sum_{\rm spins} \Bigg| \sum_{\{i_1,i_3,i_5\}} \sum_{\{i_2,i_4,i_6\}} \Bigg[\sum_{V_1=W^\pm,Z,\gamma} \sum_{V_2=W^\pm,Z,\gamma} \mathcal{M}^{{\rm a},V_1,V_2}(i_1,i_2,i_3,i_4,i_5,i_6) + \sum_{V_3=Z,\gamma} \mathcal{M}^{{\rm b},V_3}(i_1,i_2,i_3,i_4,i_5,i_6) \Bigg] \Bigg|^2$$

where $\{i_1, i_3, i_5\}$ are permutations of 'incoming' fermions and $\{i_2, i_4, i_6\}$ are permutations of 'outgoing' fermions

Construction of matrix elements

The computer program

- constructs all Born matrix elements by trying all combinations of $\{i_1, i_3, i_5\}$ and $\{i_2, i_4, i_6\}$
- stays aware of the colour structure
  (they can be distinguished by the sign of the permutations)
- calculates all matrix elements for gauge-boson production
  $(\mathcal{M}^{f_i f_j \to \mathrm{W}^+\mathrm{W}^-}, \mathcal{M}^{f_i f_j \to \mathrm{ZW}^\pm}, \mathcal{M}^{f_i f_j \to \gamma\mathrm{W}^\pm}, \mathcal{M}^{f_i f_j \to \mathrm{HW}^\pm}, \mathrm{etc.})$
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Numerical results

Numerical results use the following setup:

- Cuts to separate particles from the beam
  (e.g. $p_T(l) > 20$ GeV)
- Reconstruction of gauge bosons
  ($M_Z \pm 20$ GeV)
- Recombination for collinear photons
  ($\gamma f \to \tilde{f} \Rightarrow p_{\tilde{f}} = p_f + p_\gamma$)
- Minimal invariant mass for charged leptons in the final state
  ($M_{\mathrm{inv}} > 500$ GeV)

Production of WZ pairs

$\mathrm{pp} \to l\nu_l l'\bar{l}'$				
$M(ll'\bar{l}')$ [GeV]	σ_{Born} [fb]	σ_{EW} [fb]	Δ [%]	$1/\sqrt{2L\sigma_{\mathrm{Born}}}$ [%]
500	1.729	1.601	-7.4	5.4
600	0.899	0.814	-9.5	7.5
700	0.508	0.452	-10.9	9.9
800	0.304	0.264	-13.3	12.8
900	0.190	0.161	-15.1	16.2
1000	0.123	0.102	-16.7	20.2

Production of ZZ pairs

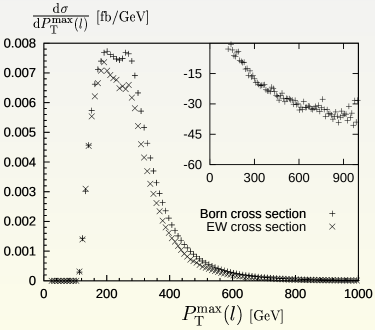
$pp \to l\bar{l}l'\bar{l}'$				
$M_{\rm inv}^{\rm cut}(l\bar{l}l'\bar{l}')$ [GeV]	$\sigma_{\rm Born}$ [fb]	$\sigma_{\rm EW}$ [fb]	Δ [%]	$1/\sqrt{2L\sigma_{\rm Born}}$ [%]
500	0.692	0.588	-15.0	8.5
600	0.356	0.291	-18.3	11.9
700	0.203	0.160	-21.0	15.7
800	0.123	0.094	-23.8	20.1
900	0.078	0.058	-26.1	25.3
1000	0.051	0.037	-28.1	31.2

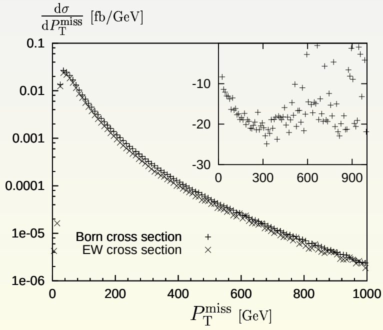
Production of W^+W^- pairs

$pp \to l\bar{\nu}_l \bar{l}'\nu_{l'}$				
$M_{inv}^{cut}(l\bar{l}')$ [GeV]	σ_{Born} [fb]	σ_{EW} [fb]	Δ [%]	$1/\sqrt{2L\sigma_{Born}}$ [%]
500	7.235	6.235	-13.8	2.6
600	3.723	3.131	-15.9	3.7
700	2.059	1.688	-18.1	4.9
800	1.201	0.959	-20.2	6.5
900	0.731	0.570	-22.0	8.3
1000	0.460	0.352	-23.4	10.4

$pp \longrightarrow W^+W^- \longrightarrow \nu_e e^+ \mu^- \bar{\nu}_\mu$

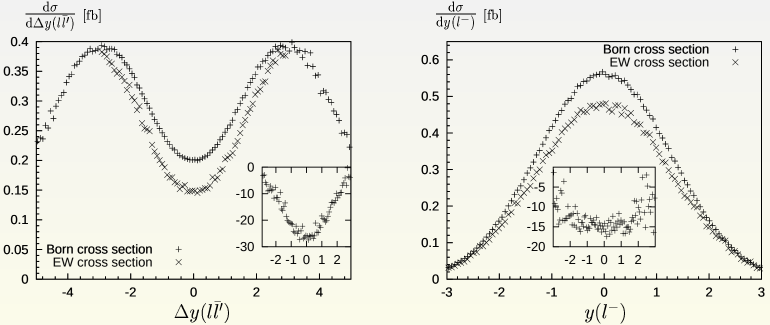
Distributions in tranverse momentum





$\mathrm{pp} \longrightarrow \mathrm{W}^+\mathrm{W}^- \longrightarrow \nu_\mathrm{e}\mathrm{e}^+\mu^-\bar{\nu}_\mu$

Distributions in rapidities



the rapidity is defined as $y = \frac{1}{2} \log \left(\frac{E+P_L}{E-P_L} \right)$

Conclusions

Our results show that

- EW corrections can reach magnitudes up to 30%.
  We found corrections of

WZ :	7 - 22%
ZZ :	15 - 28%
WW :	14 - 24%

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- EW should be taken into account in the analysis of the data at the LHC

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