Testing the Standard Model with most accurate Muon g-2 measurements

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on behalf of Muon g-2 Collaboration

PSI Colloquium, Villingen
October 12th, 2022
Outline

• Why is Muon g-2 a good test of the SM?

• How to measure the Muon g-2?

• Improvements in Run-2/3 results

• Outlook & Conclusions
The Standard Model of Particle Physics

- All known elementary particles and interactions
- Particle properties
  - Point-like
  - Mass
  - Electric, EW, strong charge
  - Spin
- Very successful theory
  - Predictions confirmed by discoveries
  - Well tested and verified

So why do we think there is physics beyond the Standard Model?

Baryon asymmetry of the Universe: Why is there not more anti-matter? Why are we even here?

Dark matter: What is it?

Neutrinos do have mass! How heavy are they? How do they obtain mass?

The Standard Model of Particle Physics is incomplete!

Dark energy: What is it?

Gravity is completely absent in the Standard Model but dominates this picture and is key to DM!
Approaches to New Physics

Cosmology

High-energy frontier

High-precision frontier

Most precise SM measurements

Electron (g-2)

Muon (g-2)

$G_{F}f_{\pi}$

$M_{Z}$

$M_{W}$

$M_{H}$

$M_{TOP}$

Precision (in PPM)

(high precision measurement) + (high precision theory calculation) = stringent SM test
Muon magnetic moment

\[ \vec{\mu}_\mu = -g_\mu \frac{e}{2m_\mu} \vec{S} \]

- Magnetic moment
- Proportionality constant
- Spin

Potential energy

\[ U = -\vec{\mu} \cdot \vec{B} \]

Torque \( \rightarrow \) precession

\[ \vec{M} = \vec{\mu} \times \vec{B} \]
Muon magnetic moment

\[ \vec{\mu}_\mu = -g_\mu \frac{e}{2m_\mu} \vec{S} \]

magnetic moment  proportionality constant  spin

Dirac (bare lepton)  \( g=2 \)

1928
Muon magnetic moment

\[ \vec{\mu}_\mu = -g_\mu \frac{e}{2m_\mu} \vec{S} \]

- magnetic moment
- proportionality constant
- spin

Dirac (bare lepton)
- \( g=2 \)
- 1928

Schwinger
- \( \alpha/2\pi \)
- 1947

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PRL 126, 141801 (2021)
Muon magnetic moment

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- Magnetic moment
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Dirac (bare lepton) 
\( g=2 \)

1928

Anomalous magnetic moment

BSM physics

\[ \Delta a_1^{\text{BSM}} \propto \frac{g_{\text{BSM}}}{16\pi^2} \frac{(\text{lepton mass})^2}{(\text{new particle mass})^2} \]

PRL 126, 141801 (2021)
Standard Model Calculation

- **QED:** perturbative approach known to 5-loop level
- **EW:** perturbative approach known to 2-loop level
- **Hadronic Vacuum Polarization (HVP)**
  - dispersive approach
  - lattice approach
  - tension
- **Hadronic Light-by-light (HLbL)**
  - dispersive approach
  - lattice approach
  - agreement

\[ a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{Had}} \]
Dispersive Approach

Dispersion Relation
follows from causality

Optical Theorem
follows from unitarity of scattering matrix

\[ a_{\mu}^{\text{had},\text{LO}} = \frac{m_{\mu}^2}{12\pi^3} \int_{s_{\text{th}}}^{\infty} ds \frac{1}{s} \hat{K}(s) \sigma_{\text{had}}(s) \]

1/s weight \( \rightarrow \) low energies most important
\( \pi^+ \pi^- \) contribute 73% to LO
need to know total hadronic cross-section \( \sigma_{\text{had}}(s) \)

Credit: Thomas Teubner
Lattice QCD Approach

• First principle calculation to predict $a_\mu$
• Numerical integration on finite space-time lattice $\rightarrow$ very computing intensive

Fist prediction with <1% uncertainty by BMW

• So far, no other high-precision $a_\mu$ from separate group

Lattice results not included in TI white paper due to low precision

• Ongoing cross checks by many groups at different energies

• Preliminary agreement in intermediate energy window
Experimental Status

- $a_\mu$(Run-1) = 0.00 116 592 040(54) [463 ppb]
- $a_\mu$(Run-2/3) = 0.00 116 592 057(25) [215 ppb]
- $a_\mu$(FNAL) = 0.00 116 592 055(24) [203 ppb]
- $a_\mu$(Exp) = 0.00 116 592 059(22) [190 ppb]

- Excellent agreement with BNL and Run-1
- BNL, Run-1 and Run-2/3 statistics dominated
- Assume 100% correlated systematics
- World experimental average dominated by FNAL
Current Status

- Tension between dispersive and lattice approach on theory side
  - Lattice results needs independent confirmation

- Long standing tension between dispersive approach and experiment reaches 5σ

- New cross-section results from CMD-3 adds a big puzzle in dispersive approach

- Very dynamic theory situation
  - Lot of efforts on lattice and dispersive approach ongoing
  - New prediction expected before Run-4/5/6

→ no conclusion about SM test

The CMD-3 point is simply a visual exercise. It is not an updated SM prediction of $a_\mu$. 
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Muon in homogeneous magnetic field

Cyclotron Motion

Centrifugal force = Lorentz force

\[ \tilde{\omega}_C = -\frac{e}{m\gamma} \tilde{B} \]

Spin Precession

Magnetic moment and field couple

\[ \tilde{\omega}_S = -g \frac{e}{2m} \tilde{B} - (1 - \gamma) \frac{e}{\gamma m} \tilde{B} \]
Muon in homogeneous magnetic field

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anomalous spin-precession frequency

\[ \omega_s - \omega_c = g_s \frac{e}{2m_s} B - \frac{e}{m} B = \frac{g_s - 2}{2} \frac{e}{m_s} B \]

anomalous magnetic moment

\[ \omega_s - \omega_c = \omega_a = a_{\mu} \]

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Muon in homogeneous magnetic field

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\[ \tilde{\omega}_S = -g \frac{e}{2m} \tilde{B} - (1 - \gamma) \frac{e}{\gamma m} \tilde{B} \]

\[ \omega_s - \omega_c = \omega_a \]

anomalous spin-precession frequency

anomalous magnetic moment

\[ \omega_a = \frac{g_\mu}{2m_\mu} B - \frac{e}{m} B = \frac{g_\mu - 2}{2} \frac{e}{m_\mu} B \]
Relativistic muon in magnetic & electric fields

\[ \tilde{\omega}_a = \tilde{\omega}_s - \tilde{\omega}_c = \frac{e}{m} \left[ a_\mu \vec{B} - a_\mu \left( \frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right] \]

- non-relativistic limit
- electron motion non-perpendicular to magnetic field
- cyclotron motion assumed motion perpendicular to magnetic field
- relativistically generated motional magnetic field proportional to electric field
- pitch of electron

\[ a_\mu^{SM} = 116591810(43) \times 10^{-11} \]

disappears for \( \gamma \approx 29.3 \)

*magic momentum*

\[ p_\mu = 3.094 \text{ GeV/c} \]
Extracting $a_\mu$

Anchor $B$, $e$ and $m_\mu$ to other high-precision measurements and calculations

$$a_\mu = \frac{\omega_a m_\mu}{\tilde{B} e}$$

We measure this ratio

$$\tilde{B} = \frac{\hbar \omega'_p}{2 \mu'_p}$$

proton spin-precession (NMR)

$$\frac{\mu_e (H)}{\mu_e}$$

Bound state QED calculation
exact
Rev. Mod. Phys. 88, 035009 (2016)

$$\frac{g_e}{2}$$

Measurement
0.13 ppt uncertainty
Phys. Rev. Lett. 130, 071801 (2023) / CODATA

Muonium hyperfine splitting
22 ppb uncertainty

10.5 ppb uncertainty at $T_r = 34.7^\circ C$
Metrologia 13, 179 (1977)

Mu-Mass

Goal: 1 ppb

total uncertainty from external quantities
25 ppb
Technique developed over 60 years

- Measured $g_\mu$ from muon at rest
- Storage ring technique to measure $g-2$
- Magic momentum technique
- NMR technique
- Superconducting storage ring magnet
- Muon Injection and magnetic kicker
- Superconducting inflector magnet

Goal: 100ppb statistical $\pm$ 100ppb systematic uncertainty
Muon Campus at Fermilab

- Accelerating protons to 8 GeV
- Form 8 bunches
- Pion production in fixed target
- Pion decay to muons (95% polarization)
- Muons outrun protons
- Muon g-2 experimental hall
Muon Campus at Fermilab

M. Fertl
R. Reimann

Momentum selective beam line

\[ p = 3.094 \frac{GeV}{c} \pm 2\% \]
~10,000 \( \mu^+ \) at 3.1 GeV
every 10 ms and 120 ns bunch length
The superconducting storage ring

- $p_{\mu}^{\text{magic}} = 3.094 \frac{GeV}{c} \pm 0.5\%$
- 3 cryostats with 4 superconducting coils (5300 A)
- 1.45 T vertical magnetic field
- 90 mm muon storage region
- 180 mm gap for vacuum chambers
- muon cyclotron period 149 ns (~6.7 MHz)
Beam Injection

- Inflector magnet cancels B field in iron yoke
- Muon can travel straight & enter the ring

field free region
How to get the beam onto storage orbit?

- Change field locally by 2% within ~150 ns
- 3 pairs of plates at roughly 90°
- Apply HV pulse at 4700 A into ~12.5 Ω in 150 ns
Keeping the muons stored

- At magic momentum electric fields have a very small impact on $\omega_a$
- Electrostatic quadrupoles focus beam vertically
- Electrostatic quadrupoles defocus beam radially
- Magnetic field focus beam radially → Complex beam dynamics
- Quasi-penning trap cover 43% of the ring
- Pulsed “electrostatic” quadrupoles
Spin projection detection

S = 1/2
right-handed

S = 1/2
left-handed

S = 1/2
right-handed

Muon decay described by weak force $\Rightarrow$ parity violation

Maximum positron energy $\cong 52.8$ MeV

Positron emitted preferably in direction of muon spin!

Figure credit: K.S. Khaw, PhD thesis, ETHZ, 2015
Muon decay in rest frame

Angular differential decay distribution is energy dependent

\[ N_e(\theta, E_e) \propto 1 - A(E_e) \cos \theta \]

\[ A(E_e) = \frac{E_{e\text{max}}^3 - 2E_e^3}{3E_{e\text{max}}^3 - 2E_e^3} \]

Figure: L. Roberts and W. Marciano, Lepton Dipole Moments

Figure credit: K.S. Khaw, PhD thesis, ETHZ, 2015
g-2 experiment with muon at rest

Decay positron detectors
- energy resolving
- segmented

Arrival time histogram at each detector will be modulated at:

$$\omega_L = g \frac{eB}{2m_\mu}$$

Measure magnetic field

Only determines $g$ not $(g-2)/2$
Spin projection detection

muon rest frame

rest frame

laboratory frame

1/12 of the ring

time & energy

beam profile

from Kim Siang Khaw
Positron detection

- 24 calorimeter stations
- 9 x 6 arrays of PbF2 crystals (Cherenkov detectors!)
- Individual SiPM readout boards
Tracking detectors

- Two tracking stations, each with 8 modules
- 128 gas-filled straws per module
- Determine $e^+$ trajectory to decay position and extrapolate to find muon beam distribution!
- Input for beam dynamics simulations
Extracting $a_\mu$

Anomalous spin precession frequency

Muon beam dynamics corrections

Clock blinding

$$\frac{\omega_a'}{\tilde{\omega}_p} = \frac{f_{\text{clock}} \omega_a^{\text{meas}}}{f_{\text{calib}}} (1 + C_e + C_p + C_{ml} + C_{pa} + C_{dd}) = \frac{f_{\text{calib}} M(x, y, \phi) \omega'_p(x, y, \phi)}{f_{\text{calib}}} (1 + B_k + B_q)$$

Spatial muon distribution

Magnetic field calibration
Spatial distribution of magnetic field
Transient magnetic fields
Fitting the “wiggle” plot

\[ f(t) \propto \langle N \rangle_{\text{thresh}} e^{-\frac{t}{\gamma \tau}} \left[ 1 + \langle A \rangle_{\text{thres}} \cos(\omega_0 t - \langle \phi \rangle_{\text{thres}}) \right] \]

Any time dependent phase shift will bias the frequency

19 different analysis from 7 independent groups
Account for complex beam dynamics \( \sim 27 \) free parameters in fit
Beam dynamics corrections

- Electric field correction
- Finite momentum distribution, not all at magic momentum
- Debunching
  - determine momentum distribution
  - determine equilibrium radius distribution

\[ C_e = -2n(1 - n)\beta^2 \frac{\langle x_e^2 \rangle}{R_0^2} \]
Magnetic field tracking

**Trolley system**
17 NMR probes
pulled through ring every ~3 days
measures spatial field dist. in storage region

**Fixed probe system**
72 azimuthal location (stations)
tracks field drift 24/7
measures field differences (drift)
Magnetic field tracking
Spatial distribution described by multipole expansion

- time interpolation using fixed probe data
Muon weighted magnetic field

- We need the field seen by the muons
- Tracking magnetic field multipole moments
- Muon distribution given by tracker data and beam dynamics simulation

\[
\frac{\omega_a}{\omega_p'} = \frac{f_{\text{clock}} \omega_a^{\text{meas}} (1 + C_c + C_p + C_{ml} + C_{pa})}{f_{\text{calib}} (M(x, y, \phi) \omega_p'(x, y, \phi)) (1 + B_k + B_q)}
\]
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Where to improve?

Total uncertainty 462 ppb

Systematic uncertainty 157 ppb

Radius: uncertainty
Area: variance
Statistics

Running Conditions

Systematic Measurements & Studies

Analysis Improvements
Where to improve?

Total uncertainty 462 ppb

Radius: uncertainty
Area: variance

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Statistical Error [ppb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run-1</td>
<td>434</td>
</tr>
<tr>
<td>Run-2/3</td>
<td>201</td>
</tr>
<tr>
<td>Run-1 + Run-2/3</td>
<td>185</td>
</tr>
</tbody>
</table>

Weighted $e^+$ in our final fit after quality control ($E > 1$ GeV, $t > 30$ us)

Last update: 07-31-2023; Total statistics = 85.2 (billions)
Running Conditions: Damaged Quad Resistors

- 2 out of 32 resistors damaged in quad plates $\rightarrow$ unstable beam storage
- Redesigned and replaced before Run-2

- Reduces phase acceptance uncertainties 75ppb $\rightarrow$ 13 ppb
- Beam oscillation frequencies become also more stable
Running Conditions: Kicker Strength

Upgraded kicker cables while Run-3 so kicker can run at nominal strength

- Muon distribution more centered
  - All systematics related to asymmetric beam shape reduced

- Muon momentum distribution better centered on magic momentum
  - E field correction reduced

- Phase space matching improved
  - Smaller beam oscillations
Magnetic field quadrupole transients

Pulsing electrostatic quadrupoles for beam confinement leads to magnetic field transient. Not seen by Fixed Probes: Fast transient fields are shielded by aluminum in vacuum chambers.

Special NMR probe with PEEK housing allows to measure effect in storage volume.
Magnetic field quadrupole transients

Run 1
- Limited measurement points
- Large uncertainty: 92 ppb

Run 2/3
- Probe movable on trolley rails
- Detailed measurement campaign over > 1 month
- Uncertainty reduced to 20 ppb
Statistics

Running Conditions

Systematic Measurements & Studies

Analysis Improvements
Analysis Improvements: Pile-up

- 2 $e^+$ arriving at same time can be mistaken for 1
- Rate dependent $\rightarrow$ can bias $\omega_a$
- Reduced uncertainty by:
  - Improved **reconstruction**
  - Improved **correction algorithm**

**Phase of high-energy muon**

$\neq$

**Phase of two low-energy muons**
Putting all together

- Uncertainty reduced by factor >2
- Statistic and systematic uncertainty reduced by similar amount
- Systematic uncertainty below TRD goal
- Still statistics dominated

Run-1

Total uncertainty: 462 ppb

- 434 ppb
- 157 ppb

Run-2/3

Total uncertainty: 215 ppb

- 201 ppb
- 70 ppb

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Radius: uncertainty
Area: variance
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Outlook

Last update: 10-11-2023; Total statistics = 322.1 (billions)

**Muon g-2 (FNAL)**

- **Run6 endend on 9th July 2023**
- Exceeded TRD goal of 21 BNL statistics
- Last part of run 6 dedicated to systematic studies
- Further field studies ongoing until early 2024
Outlook

Muon g-2 (FNAL)

Last update: 10-11-2023; Total statistics = 322.1 (billions)

- Run-6
- Run-5
- Run-4
- Run-3
- Run-2
- Run-1

Data fully processed

Analysed positrons (AMethod) [billions]

ωa statistical precision [ppb]

0 100 200 300 400 500
0 70 140 210 280 350

- Published

Analysis ongoing
Expect publication in 2025

Much more statistics
Another factor of about 2 in statistic uncertainty
Further improvements

Improved Running Conditions

- Quadrupole RF system (Run-5/6)
  - Reduced horizontal beam oscillations

Systematic Measurements & Studies

- New detectors (scintillating fibers)
  - For direct beam measurements (Run-6)
- Better understanding and modeling of beam dynamics
Conclusions

• High precision measurements of Muon g-2 stringent test on SM theory

• Run-2/3 data consistent with Run-1 and BNL

• Improvement by factor >2 in statistical and systematic uncertainty

• Surpassed TRD goals in statistics and systematics

• First time a three-way comparison of $a_\mu$ is possible
  • Dispersive-approach lattice approach, experiment
  • Very interesting

• Another reduction by factor of 2 in statistical uncertainty from Run-4/5/6

• Analysis for EDM, DM and LV on-going as well
Thank you for your attention
181 collaboration members worldwide

**US Universities**
- Boston
- Cornell
- UIUC
- James Madison
- Kentucky
- Massachusetts
- Michigan
- Michigan State
- Mississippi
- North Central College
- Regis
- Virginia
- Washington

**US National Labs**
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