String Theory and the AdS/CFT correspondence

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Generalities

What string theory is (and what it is not)?

It is a promising approach towards a quantum theory incorporating high energy physics and general relativity.

However, it is not a `complete' theory: there are many conceptual problems (in particular its off-shell formulation) that are not well understood.

Generalities

What string theory is (and what it is not)?

There are also (mainly technical) problems in understanding string theory for time-dependent and cosmological configurations, in particular for de Sitter backgrounds.

On the other hand, string theory seems to provide, for example, a convincing explanation of black hole entropy in terms of microscopic states (at least for certain black holes).

Generalities

What string theory is (and what it is not)?

String theory makes very specific and testable predictions…

However, as for any theory of quantum gravity, these predictions are only `sharp' near the Planck scale, and direct experimental verification seems currently out of reach.

Generalities

As a consequence, supporting evidence for string theory has appeared more indirectly, e.g.

- AdS/CFT correspondence has led to testable insights into conventional QFTs
- Triggered new developments in mathematics, e.g. mirror symmetry.
- Inspiration for model building, e.g. higher dimensions, supersymmetry, etc.
- Given rise to new techniques for perturbative and non-perturbative analysis of (susy) QFTs, e.g. Seiberg-Witten, etc.

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String basics

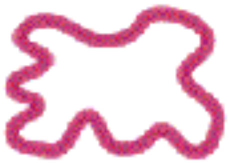
The starting point of string theory is rather unconventional (and certainly not what you would begin with if you were to try and quantise gravity).

[In fact, the history of the subject is rather convoluted, and it only become clear at some later stage that string theory actually includes gravity.]

String basics

The basic idea is that the fundamental objects, in terms of which the theory is being formulated, are one-dimensional strings.

These strings can either be



or

closed

open

String basics

They propagate (initially) in a given fixed background.

The consistency conditions (conformal beta equations) require that this background satisfies a modification of the

Einstein equations

Thus string theory incorporates general relativity.

String basics

Furthermore, the excitation spectrum
of the string contains the `graviton‘ that
describes fluctuations of the background.

Thus string theory may in fact be
background independent.

String basics

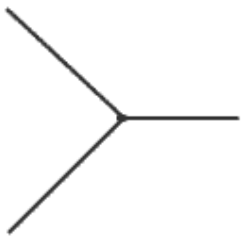
The other vibrational excitations of the string contain the quanta that correspond to the familiar elementary particles.

Thus string theory incorporates
(some extension of) the

standard model of particle physics.

String basics

instead of



This smooth description suggests that string theory has better UV properties than conventional quantum field theories.

[On the other hand, string field theory is not that well developed. In particular, one does not yet understand how to derive the `Feynman rules' of string field theory from first principles.]

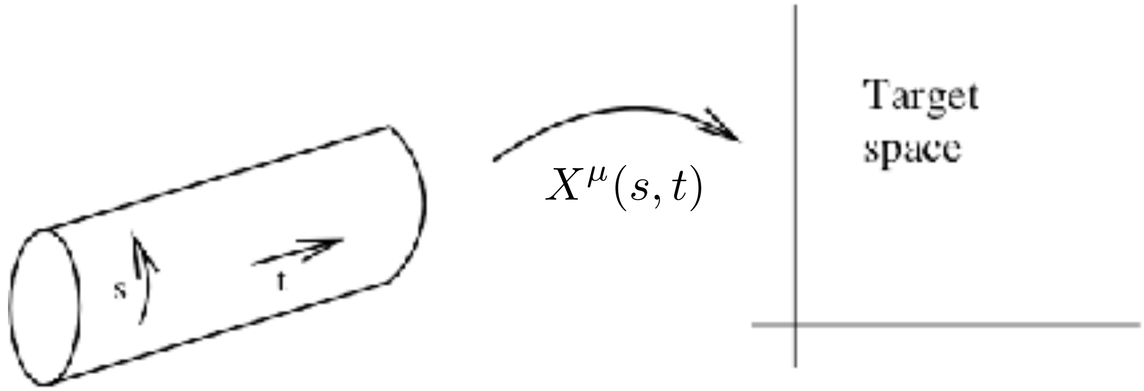
String basics

So far I have described the standard string folklore….

But how does one actually describe string theory explicitly?

Quantitative strings

In order to describe strings quantiatively, think of them in terms of a 2-dimensional field theory that is defined on the world-sheet of the string.



Sigma model

The propagation of the string is described by specifying to which point in target space (space-time) every point on the world-sheet is mapped to.

This map is controlled by the sigma-model action

$$S = \frac{T}{2} \int ds\, dt \sqrt{h}\, h^{mn} \partial_m X^\mu \partial_n X^\nu G_{\mu\nu}$$

world-sheet metric

space-time metric

Sigma model

The sigma model is classically invariant under reparametrisations and Weyl rescalings of the metric.

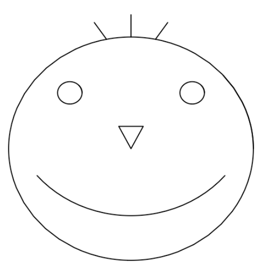
Can use this symmetry to go to `conformal gauge': take world-sheet metric to be proportional to usual 2d Minkowski metric.

Then the residual symmetry is the conformal (Weyl) symmetry — the resulting 2d field theory is therefore a conformal field theory.

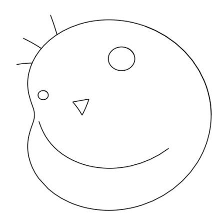
Conformal transformations

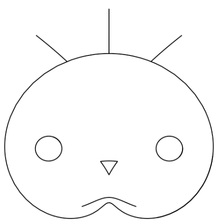
The conformal transformations preserve the metric up to rescalings. They therefore preserve angles, but not necessarily lengths.

For a conformally invariant theory it therefore does not matter whether one looks like this



or like





Virasoro algebra

The conformal symmetry in 2 dimensions is thus very powerful!

In fact, the algebra of infinitesimal conformal transformations is infinite dimensional:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m,-n}$$

[Virasoro algebra]

c: central charge

Conformal symmetry

Because of this large symmetry, 2d conformal field theories can be solved essentially based on symmetry considerations alone.

[In fact, 2d conformal field theories have a very rich mathematical structure: they define vertex operator algebras, and have had a significant impact on many areas of modern mathematics, such as group theory, number theory, algebra, etc.]

Critical dimension

From the point of view of string theory, the conformal symmetry is a gauge symmetry. Just as in electrodynamics, this gauge symmetry can then remove the negative norm states (ghosts) that the covariant theory initially has.

In the present context, this requires that

$$c \leq 26 \ .$$

[Goddard, Thorn '72]

Critical dimension

If the target space is flat then c=dimension.

Thus D=26 is the critical dimension of (bosonic) string theory.

In order to describe 4d physics, the idea is then that

$$26 = c_{\mathrm{4d}} + c_{\mathrm{int}} \ , \qquad\qquad c_{\mathrm{int}} = 22$$

i.e. 22 dimensions are compactified on some internal manifold — extra dimensions.

Compactification

The `internal’ theory may again be defined in terms of a non-linear sigma model, but in general it may be any conformal field theory with $c=22$.

It is therefore not really appropriate to talk about an `internal manifold’ — the relevant conformal field theory may or may not have a geometric interpretation.

Fermions

The analysis for the (world-sheet) fermionic string is similar: in this case the relevant gauge symmetry is the N=1 superconformal symmetry that contains in addition to the Virasoro generator a superfield G

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m,-n}$$

$$[L_m, G_r] = (\frac{m}{2} - r)G_{m+r}$$

$$\{G_r, G_s\} = 2L_{r+2} + \frac{c}{3}(r^2 - \frac{1}{4})\delta_{r,-s} \ .$$

No Ghost Theorem (II)

In this case the no-ghost theorem requires that

$$c \leq 15 \ .$$

[Goddard, Thorn '72]

Since each space-time direction contributes

$$c = 1 + \frac{1}{2} = \frac{3}{2} \qquad \Longrightarrow \qquad D = 10 \ .$$

critical dimension of superstring

Supersymmetry

The fermionic string is not necessarily space-time supersymmetric, not even in D=10 dimensions.

[Thus it is not really true that `string theory predicts space-time supersymmetry'.]

However, many of the most interesting (and best understood) string theories are space-time supersymmetric.

Space-time supersymmetry

Spacetime supersymmetry actually requires that the world-sheet symmetry is the

N=2 superconformal algebra

generated by: Virasoro
u(1) Kac-Moody algebra
2 supercharge generators

Geometry

For theories with N=2 superconformal symmetry can at least partially identify the corresponding `geometry'.

Topological twist: each supercharge generator is nil-potent and defines a cohomology.

$\updownarrow$

[Lerche, Vafa, Warner '89]
[Witten '98]

can be identified with the cohomology of the corresponding geometry!

Mirror Symmetry

However, since one may take either of the two supercharges, a given N=2 conformal field theory typically gives rise to (two) different geometries:

Mirror symmetry

[Candelas et.al. '91]

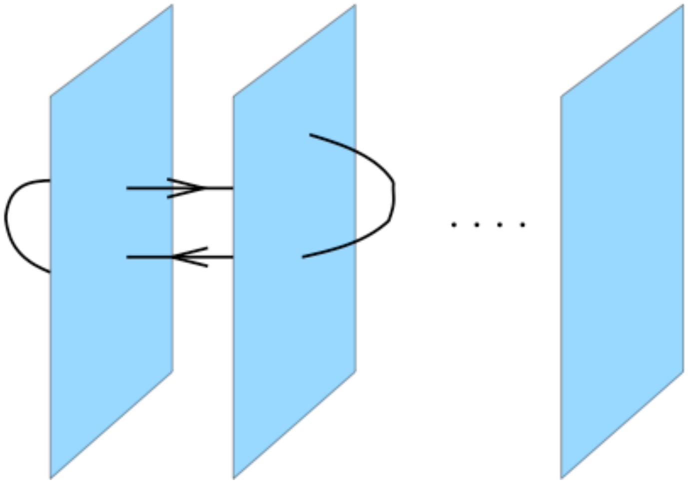
D-branes

So far have mainly considered closed string theory.
Open strings can be introduced by considering D-branes:

[Polchinski ’95]

D-branes

If we consider a `stack' of D-branes, the open string degrees of freedom give rise to a Yang-Mills theory living on the world-volume of the brane:

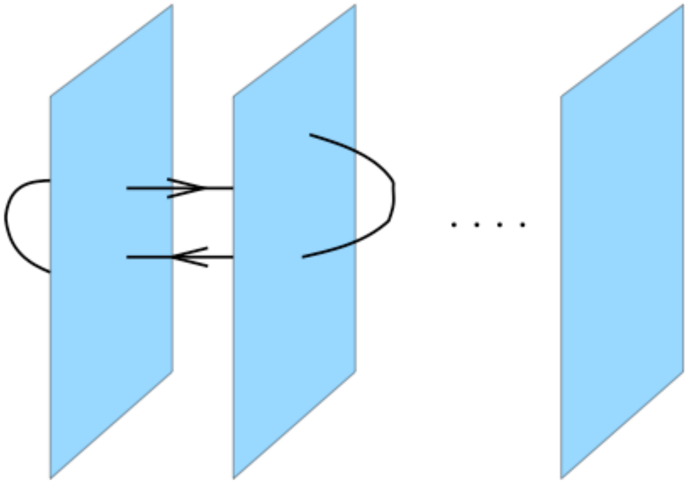


N D-branes:

[S]U(N) gauge theory.

D-branes

On the other hand, the D-branes also have an effect on the background geometry — they are, in particular, sources for the gravitational field…



N D-branes:

AdS geometry!

AdS/CFT duality

[Maldacena '97]

AdS / CFT duality

For example, for the case of N D3 branes one finds (in the large N limit)

superstrings on
$\mathrm{AdS}_5 \times S^5$

=

SU(N) super Yang-Mills theory in 4 dimensions

4d non-abelian gauge theory similar to that appearing in the standard model of particle physics.

Relation of parameters

The relation between the parameters of the two theories is

$$\left(\frac{R}{l_{\mathrm{Pl}}}\right)^4 = N \qquad g_{\mathrm{string}} = g^2_{\mathrm{YM}} \qquad \left(\frac{R}{l_{\mathrm{s}}}\right)^4 = g^2_{\mathrm{YM}} N = \lambda$$

AdS radius in Planck units

AdS radius in string units

‘t Hooft parameter

Strong weak duality

For example, in the large N limit of gauge theory at large ‘t Hooft coupling

$$\left(\frac{R}{l_{\rm Pl}}\right)^4 = N \qquad g_{\rm string} = g_{\rm YM}^2 \qquad \left(\frac{R}{l_{\rm s}}\right)^4 = g_{\rm YM}^2 N = \lambda$$

large

small

large

Supergravity (point particle) approximation is good for AdS description.

AdS/CFT duality

This is interesting since it gives insights into strongly coupled gauge theories using supergravity methods.

Many applications and insights:

- anomalous dimensions in N=4 SYM
  [Minahan,Zarembo,Beisert,Staudacher,...]
- structural insights into amplitudes
  [Witten,Cazacho,Arkani-Hamed,Alday,Maldacena, Korchemsky,Drummond,Sokatchev,...]
- quark gluon plasma
  [Liu,Rajagopal,Wiedemann,Gubser,...]
- quantum critical systems
  [Hartnoll,Herzog,Horowitz,Kachru,Sachdev,Son, Erdmenger...]
- ...

Strategy for a proof

There is very good evidence that at least this supersymmetric version of the duality is correct. However, we have not yet succeeded in proving it.

Together with Rajesh Gopakumar, I have recently made progress in this direction. Basic idea: consider

- **Tensionless regime**
- **Lower dimensional version (AdS3)**

Weakly coupled gauge theory

The tensionless regime arises in another corner of parameter space where the gauge theory is weakly coupled

$$\left(\frac{R}{l_{\mathrm{Pl}}}\right)^4 = N \qquad g_{\mathrm{string}} = g_{\mathrm{YM}}^2 \qquad \left(\frac{R}{l_{\mathrm{s}}}\right)^4 = g_{\mathrm{YM}}^2 N = \lambda$$

large

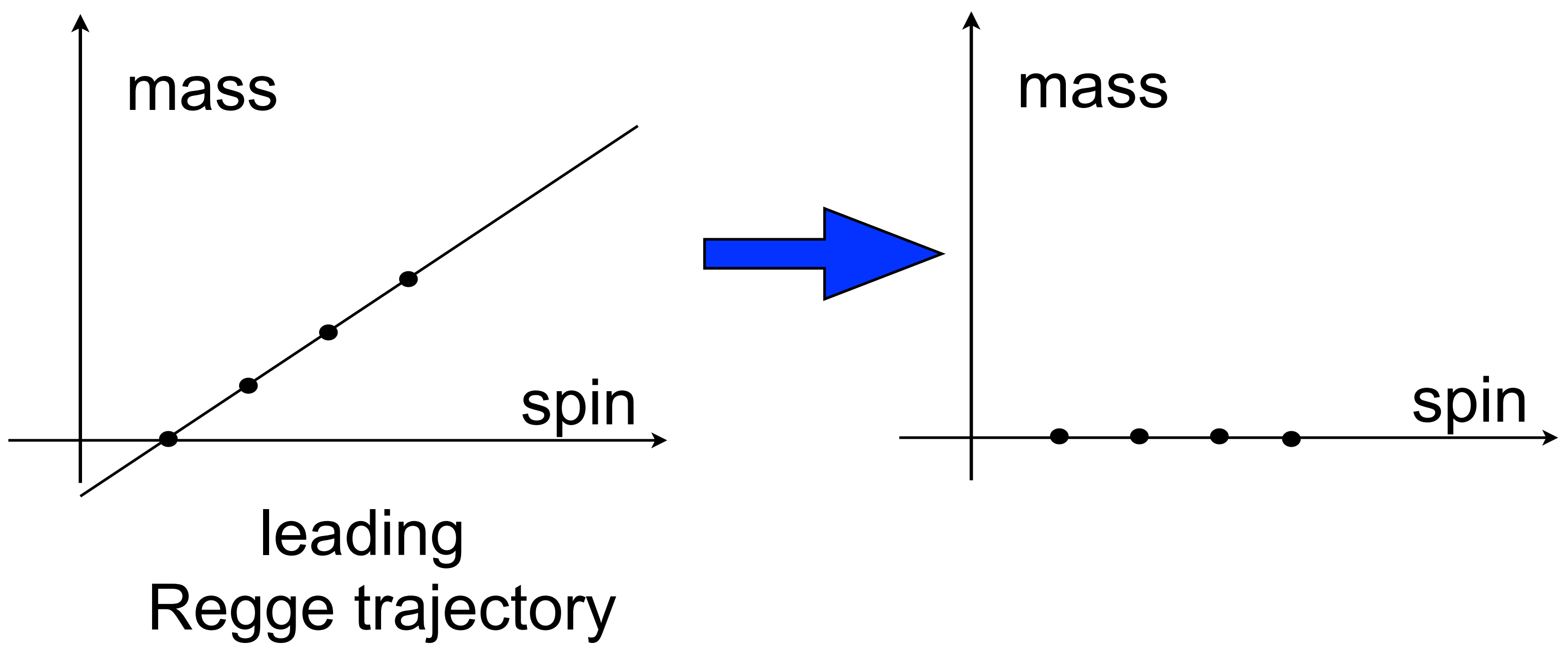
small

$l_{\mathrm{s}} \to \infty$ `tensionless strings'

[Sundborg '01] [Witten '01]
[Sezgin,Sundell '01]

Tensionless limit

In tensionless limit all string excitations become massless:



Higher spin theory

Resulting theory has an infinite number of massless higher spin fields, which generate a very large gauge symmetry.

$\rightarrow$ effective description in terms of Vasiliev Higher Spin Theory.

maximally symmetric/unbroken phase of string theory

3d version

The situation is particularly simple for the 3d AdS case for which the dual is a 2d CFT.

- AdS3 HS theories are much simpler
- Much better control over 2d CFTs

This has allowed us to make a concrete proposal for such a HS-CFT duality.

[MRG,Gopakumar '10]

We were subsequently able to perform many precision tests (quantum symmetry, spectrum).

[MRG,Gopakumar, Hartman, Raju '11], [MRG,Gopakumar '12]

3d version

The supersymmetric version of this HS-CFT duality also suggests how to lift it to the full stringy level.

Using the fact that for AdS3 there is a solvable world-sheet theory for the description of strings, we have recently shown that the tensionless limit of this worldsheet theory is exactly dual to a specific CFT (the symmetric orbifold)

An exact AdS/CFT duality

[MRG,Gopakumar ’18]
[Eberhardt, MRG, Gopakumar ’18]

Exact AdS/CFT duality

More concretely, our arguments show that

2d CFT superstrings on

$$\mathrm{Sym}_N(\mathbb{T}^4) \quad = \quad \mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$$

1 unit of NS-NS flux

This background describes a tensionless string theory, where massless higher spin fields are present.

Exact AdS/CFT duality

$$\mathrm{Sym}_N(\mathbb{T}^4) \quad = \quad \mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$$

1 unit of NS-NS flux

Both sides are explicitly solvable and have free field realisations.

This opens the door for all sorts of quantitative tests of the (stringy) duality, and is likely to lead to a full proof of it, at least for this specific case.

[Eberhardt, MRG, Gopakumar '18 & in progress]

Summary

Have given some basic introduction into string theory:

- world-sheet description
- critical dimension (no ghost theorem)
- fermions
- spacetime supersymmetry & mirror symmetry
- open strings & D-branes
- AdS/CFT correspondence

Summary

While string theory continues to be probably the most viable candidate for a

Quantum Theory of Gravity

recent work has mostly concentrated on other aspects of the theory where **more direct contact** with **experimentally accessible** physics is possible, in particular

AdS/CFT correspondence $\longrightarrow$ insights into strongly coupled QFT

Summary

While there is good evidence that the idea of the AdS/CFT duality is rather general, we do not yet know its full range of applicability.

Therefore important to understand more fundamentally how the duality works — this is likely to require world-sheet techniques…

I have sketched some recent progress in this direction: at least in lower dimensions a detailed understanding of the duality seems now within reach.

Thank you!