Imaging Magma Chambers
Inside the Human Body

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Ultrasound waveform tomo-
graphy of a human breast
[courtesy of N. Duric, 2019]
Shear velocity at 100 km depth
from full-waveform inversion
[Fichtner et al., 2018]
Purposes of this talk

Introduction to seismic imaging of the Earth using earthquakes

Translation of seismic imaging concepts to medical ultrasound

Focus on fundamental concepts

Whetting your appetite
Numerical simulation of the Tohoku earthquake, 11 March 2011

magnitude: 9.1 [4th largest earthquake ever recorded]
liberated energy: $\sim 4 \cdot 10^{22} \text{ J}$ [70× world energy consumption, ~10²× medical ultrasound pulse]
Black Forest Observatory
Epicentral distance: 83.3°
Black Forest Observatory
Epicentral distance: 83.3°

**Data: Seismograms**

- Typical frequency range: 1 mHz – 1 Hz
- **Body waves**
  - Vertical
  - East-west
  - North-south

**Seismology & Wave Physics**
SONIFICATION: PLAYING IT 1500 TIMES FASTER
EARLY SEISMIC TOMOGRAPHY INSPIRED BY MEDICAL IMAGING [AKI ET AL., 1977]
Early Seismic Tomography Inspired by Medical Imaging [Aki et al., 1977]
compressional wave velocity between 17-36 km depth
compressional wave velocity between 17-36 km depth

Baltic Shield
[old (>2.5 Ga) and cold rocks]
Compressional wave velocity between 17-36 km depth is faster in some areas and slower in others. Volcanically active areas are indicated by young and hot rocks of the Oslo-Graben.
Multi-scale full-waveform inversion (FWI) of the whole Earth
Dynamics, evolution and composition of the Earth
# The Bigger FWI Picture

<table>
<thead>
<tr>
<th>Scale</th>
<th>Internal structure of the Sun</th>
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**Scale:**
- 1’000’000 km
- 100’s – 10’000 km

**Internal structure of the Sun**
- Dynamics, evolution and composition of the Earth

**Other applications:**
- Monitoring of the Comprehensive Nuclear-Test-Ban Treaty
- Exploration and reservoir monitoring
- Engineering applications
- Technology transfer to medical tomography and non-destructive testing

**Scale:**
- 1’000’000 km
- 100’s – 10’000 km
- 100 m – 10 km
- 10 – 100 m
- 1 – 100 cm
- 10's – 100 km
THE BIGGER FWI PICTURE

Scale

1’000’000 km
- Internal structure of the Sun
- Dynamics, evolution and composition of the Earth

100’s – 10’000 km
- Earthquake source inversion
- Reliable tsunami warnings
- Monitoring of the Comprehensive Nuclear-Test-Ban Treaty
## The Bigger FWI Picture

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The Bigger FWI Picture

- Internal structure of the Sun
- Dynamics, evolution and composition of the Earth
- Earthquake source inversion
- Reliable tsunami warnings
- Monitoring of the Comprehensive Nuclear-Test-Ban Treaty
- Exploration and reservoir monitoring
- Engineering applications
- Technology transfer with medical imaging and non-destructive testing
- ...
Part I: Seismic Full-Waveform Inversion

1. The FWI Dream
2. Simulating seismic waves in complex media
3. How wiggles see the Earth

Part II: Medical Full-Waveform Inversion

4. Motivation & challenges
5. In-vivo imaging of a mouse
6. Ultrasound imaging of the human brain
PART I

SEISMIC FULL-WAVEFORM INVERSION [FWI]
Seismograms from the 2011 Tohoku earthquake

vertical

east-west

north-south
Traveltime tomography

- Measure arrival times of few identifiable waves
- Model and invert using geometric ray theory [as in optics]
Traveltime tomography

- Measure arrival times of few identifiable waves
- Model and invert using geometric ray theory [as in optics]

😀 Fast and inexpensive
😭 Valuable information is discarded.
Full-waveform inversion

- Exploit complete recordings
- Model and invert using numerical solutions of the wave eq.
The FWI Dream

Full-waveform inversion

- Exploit complete recordings
- Model and invert using numerical solutions of the wave eq.

😊 More information >> higher resolution
😢 Methodologically complex and computationally expensive

Seismograms from the 2011 Tohoku earthquake
The FWI Dream

How can we model complete wavefields in complex media?

How do the wiggles depend on 3-D [Earth, human, ...] structure?
How can we model complete wavefields in complex media?

>> spectral-element method

How do the wiggles depend on 3-D [Earth, human, …] structure?

>> adjoint method
Principal desiderata

- Operates in complex media [irregular topography, complex shapes of human body parts]
- Flexible and realistic material properties [attenuation, anisotropy]
- Solid, fluid and solid-fluid coupled media [solid Earth, soft tissue, oceans, human head]
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SEM basics

- Consider weak form of the wave equation
- Discretise into hexahedral elements
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SEM basics

- Consider weak form of the wave equation
- Discretise into hexahedral elements
- Map elements onto unit cube
THE SPECTRAL-ELEMENT METHOD (SEM)

Principal desiderata

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SEM basics

- Consider weak form of the wave equation
- Discretise into hexahedral elements
- Map elements onto unit cube
- Integrate in space using Gauss-Lobatto-Legendre (GLL) points >> diagonal mass matrix

\[ \ddot{u} = M^{-1} \cdot (f - K \cdot u) \]

- \( \ddot{u} = \) discrete wavefield
- \( M = \) mass matrix
- \( K = \) stiffness matrix
- \( f = \) external force
Principal desiderata

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SEM basics

- Consider weak form of the wave equation
- Discretise into hexahedral elements
- Map elements onto unit cube
- Integrate in space using Gauss-Lobatto-Legendre (GLL) points >> diagonal mass matrix
- Integrate in time using an explicit method
Earth and planetary scale

10’000 km - fluid/solid

5’000 km - fluid/solid

1 km - fluid/solid

11 March 2011 Tohoku earthquake

Hypothetical marsquake

Sub-sea sedimentary layers and salt body
The Spectral-Element Method (SEM)

Engineering and medical scale

10 cm - solid

elastic wave inside a clamp

20 cm - solid

acoustic wave inside a human breast
Given sufficient computational resources we can model complete seismograms.
Given sufficient computational resources we can model complete seismograms.

How does this [or any other] wiggle see the Earth?
ADJOINT METHOD & SENSITIVITY KERNELS

- **Sensitivity kernel** >> generalised Fresnel zone
- 3-D representation of the partial derivatives of data w.r.t. medium parameters
ADJOINT METHOD & SENSITIVITY KERNELS

- Sensitivity kernel >> generalised Fresnel zone
- 3-D representation of the partial derivatives of data w.r.t. medium parameters
- **Efficiently computed using the adjoint method**
  - 1 forward simulation
  - 1 adjoint simulation [same cost as forward simulation]
- Enable iterative updating of the medium
**Adjoint Method & Sensitivity Kernels**

**PP wave** at 20 s period
sensitivity to compressional wave speed

**PcP wave** at 20 s period
sensitivity to compressional wave speed

Hung et al. 2000
... use the right type of waves [wave type, frequency, amplitudes, traveltimes, ...]

to learn what you want to know.
PART II

MEDICAL FULL-WAVEFORM INVERSION
Key Motivations

- Free of ionising radiation
- Inexpensive
- Enable frequent, repetitive analyses
- Quantitative imaging for improved diagnoses
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- Free of ionising radiation
- Inexpensive
- Enable frequent, repetitive analyses
- Quantitative imaging for improved diagnoses

Major challenges

- Small bandwidth of standard transducers
- Computational cost at high frequencies
- Complex geometry and meshing
- Fluid-solid wavefield interactions
EXAMPLE I: IN-VIVO IMAGING OF A MOUSE
Example 1: In-vivo Imaging of a Mouse

Goals

- Workflow development with realistic data
- Towards improved cancer diagnoses without human experiments
**Goals**
- Workflow development with realistic data
- Towards improved cancer diagnoses without human experiments

**Setup**
- In-vivo dataset of a mouse
- Acquired with a transmission-reflection ultrasound imaging platform [Mercep et al., 2020]
Step 1:

- Traveltime tomography using first-arriving ultrasound pulses
- Initial model for FWI
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- Traveltime tomography using first-arriving ultrasound pulses
  - Initial model for FWI

Step 2:
- FWI with successively increasing bandwidth
  - 750 kHz – 1.5 MHz
  - 750 kHz – 2.5 MHz
  - 750 kHz – 2.85 MHz
  - 750 kHz – 3 MHz
- 84 gradient descent (L-BFGS) iterations in total
EXAMPLE 1: IN-VIVO IMAGING OF A MOUSE

traveltime tomography

FWI
EXAMPLE II: IMAGING THE HUMAN BRAIN
Example II: Imaging the Human Brain

Goals

- Proof of concept. Does it work at all?
- Development of an inversion strategy.
Much of the energy is reflected off the skull.
EXAMPLE II: IMAGING THE HUMAN BRAIN

Challenges

Much of the energy is reflected off the skull.

The skull is geometrically complex and difficult to mesh.
Much of the energy is reflected off the skull.

The skull is geometrically complex and difficult to mesh.

The head is a coupled fluid-solid system.
Effect of fluid-solid interaction

The head is a coupled fluid-solid system.
Example II: Imaging the Human Brain

Synthetic, *in-silico*, FWI experiment

FWI reconstruction

MIDA phantom [Iacono et al., 2015] ground-truth model
Transferability

- Scale-independence ensures that FWI math is identical in seismic and medical imaging.
- Codes [SEM] can be transferred rather easily.

Preliminary results

- Detailed images of soft tissue using in-vivo data [mouse experiment].
- Successful in-silico experiment for acousto-elastic brain imaging.
- Promising and worth pursuing further [towards in-vivo FWI of human body parts].

Challenges

- Bandwidth of existing ultrasound devices.
- Computational cost at high frequencies.
- Cultural and regulatory barriers.
CONCLUSIONS

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Thanks for your attention!