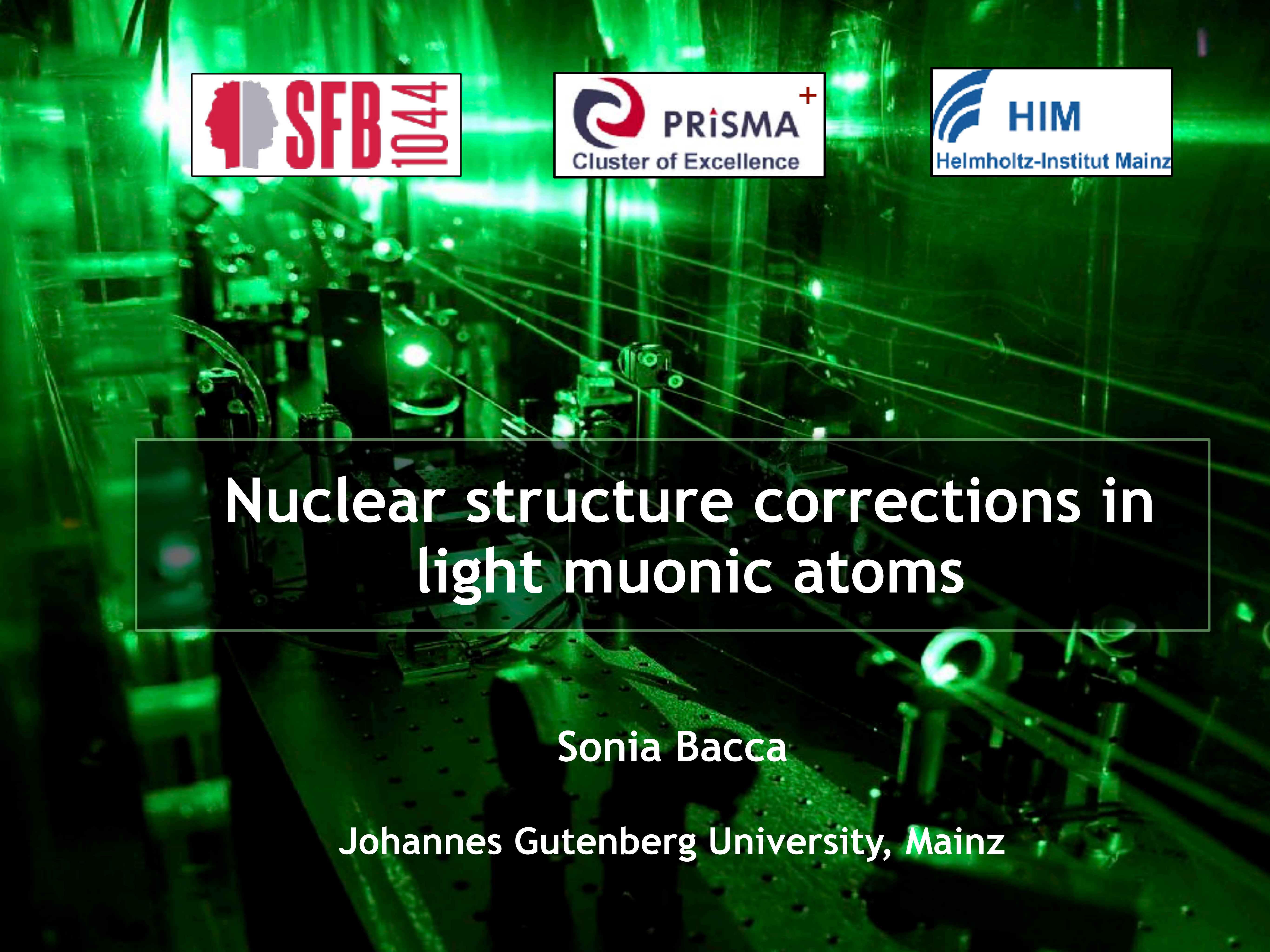


SFB 1044

PRISMA+
Cluster of Excellence

HIM
Helmholtz-Institut Mainz

Nuclear structure corrections in light muonic atoms

Sonia Bacca

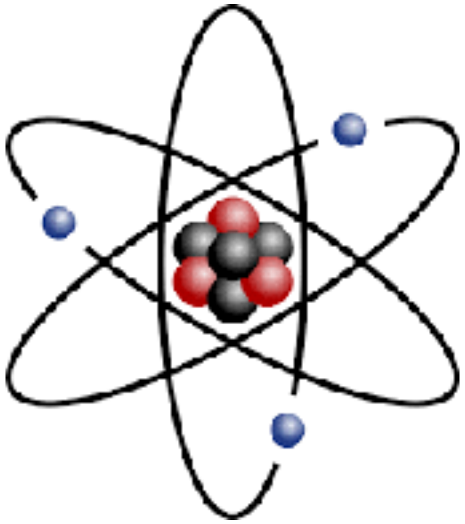
Johannes Gutenberg University, Mainz

Outline

- Muonic atoms and the Lamb shift
- Proton radius puzzle
- Theory of muonic atoms
- Ab initio nuclear theory
- Impact on the measurements
- Summary and outlook

What are muonic atoms?

Exotic atoms



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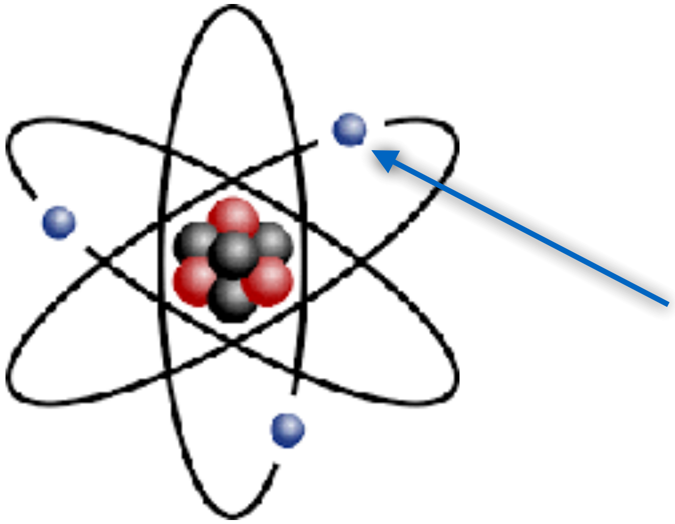
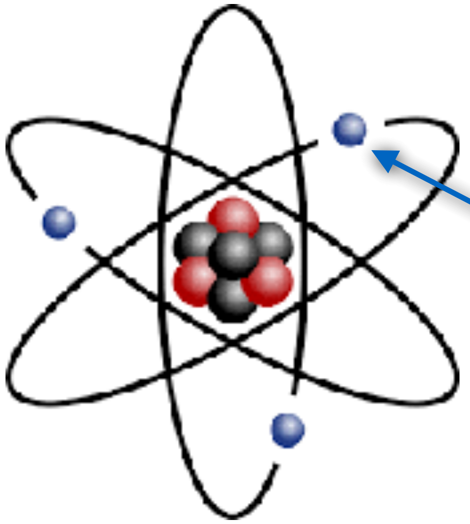


Electron replaced
with a muon

What are muonic atoms?

Exotic atoms



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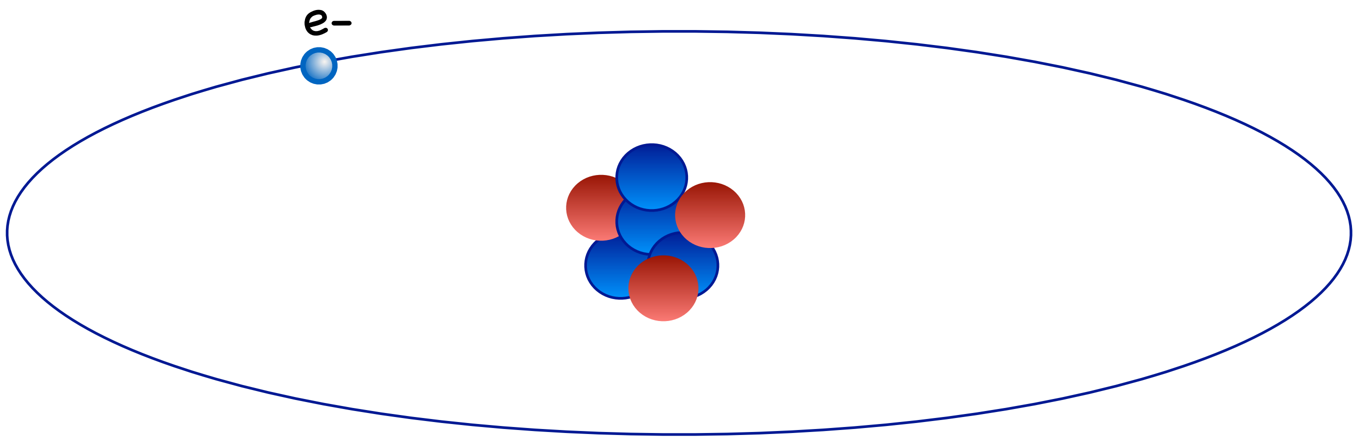
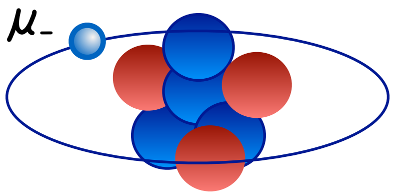
Hydrogen-like systems

Ordinary atoms



Muonic atoms



muon more sensitive to the nucleus

What are muonic atoms?

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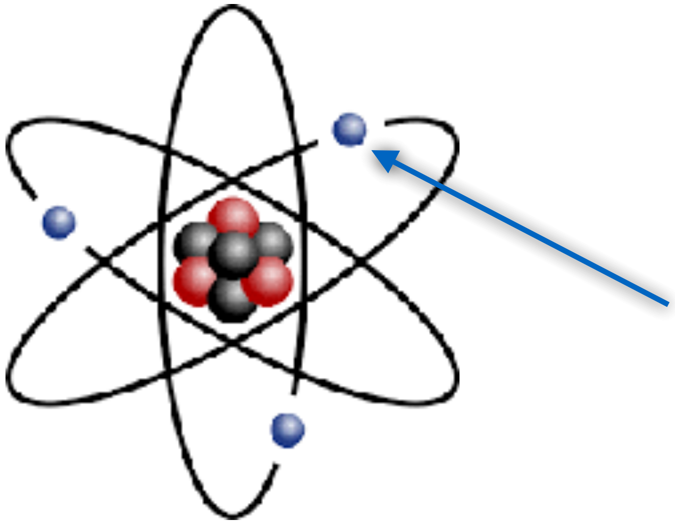

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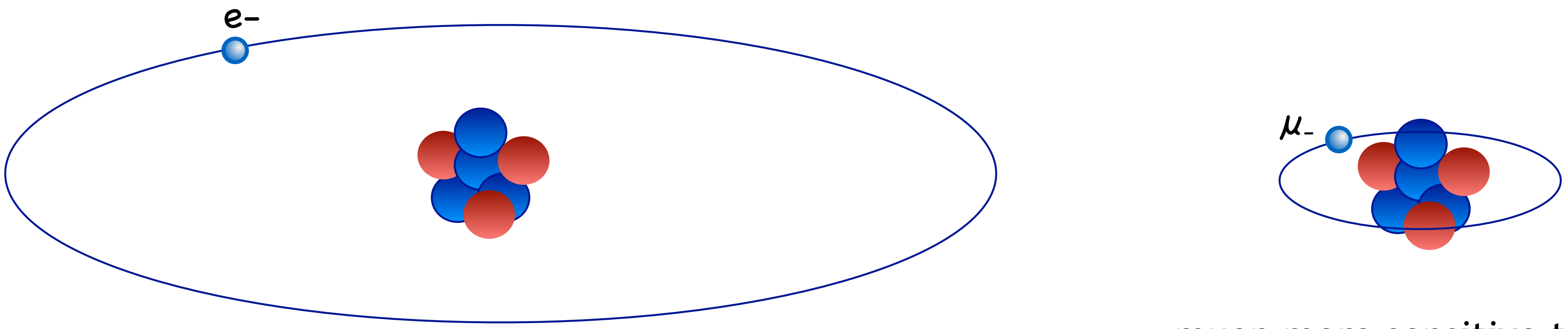
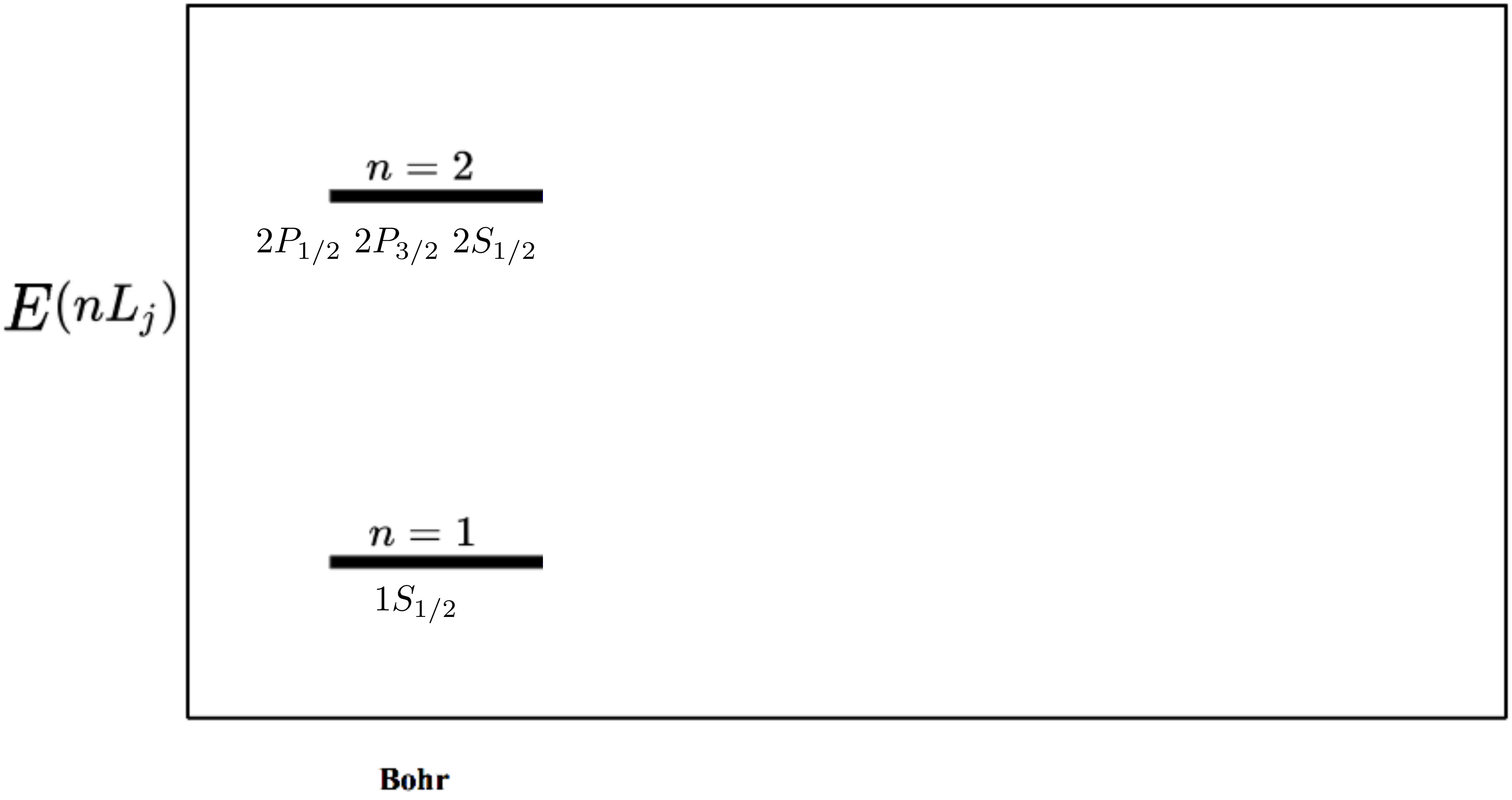


muon more sensitive to the nucleus

Can be used as a precision probe for the nucleus

Lamb Shift



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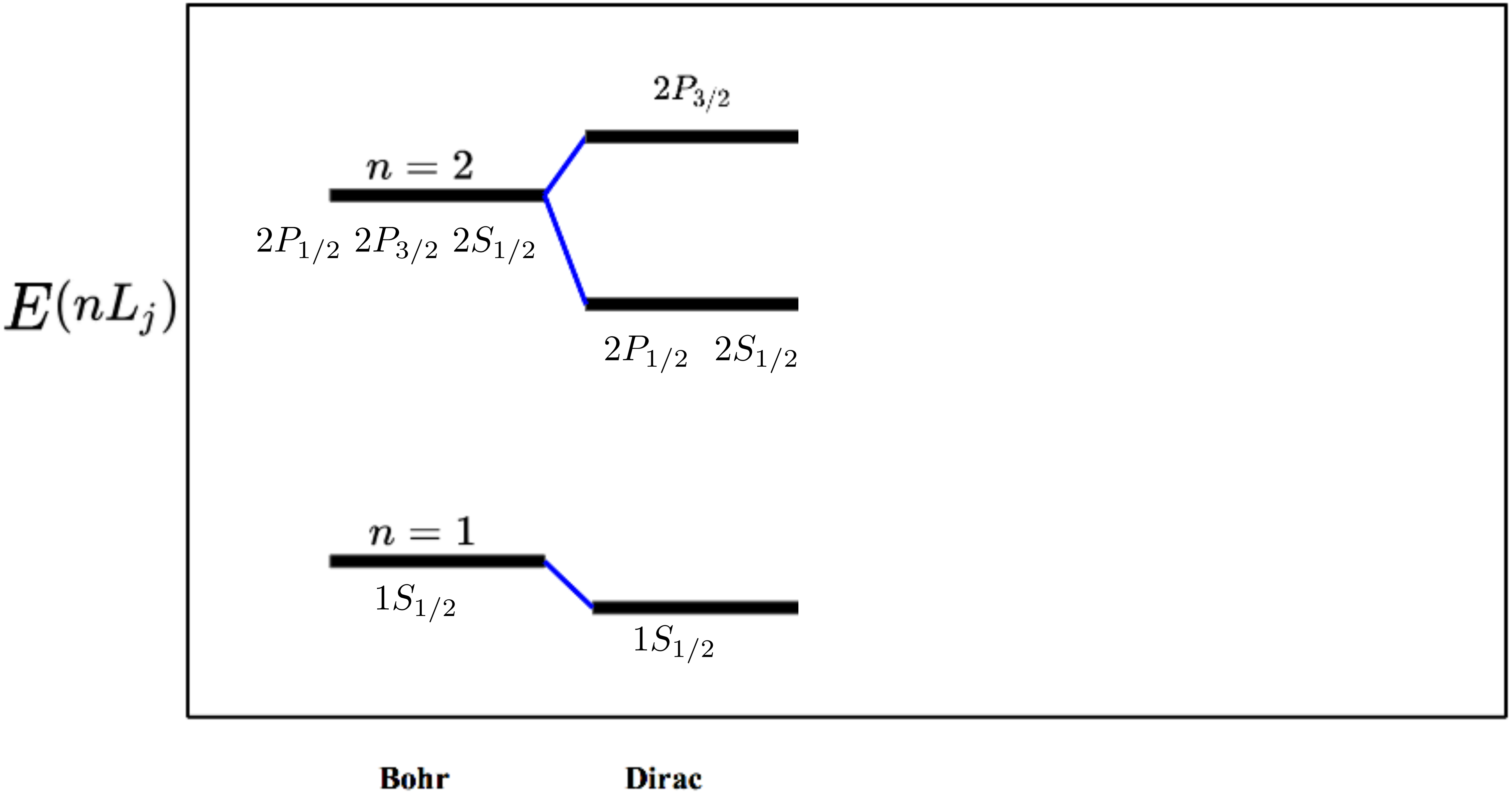
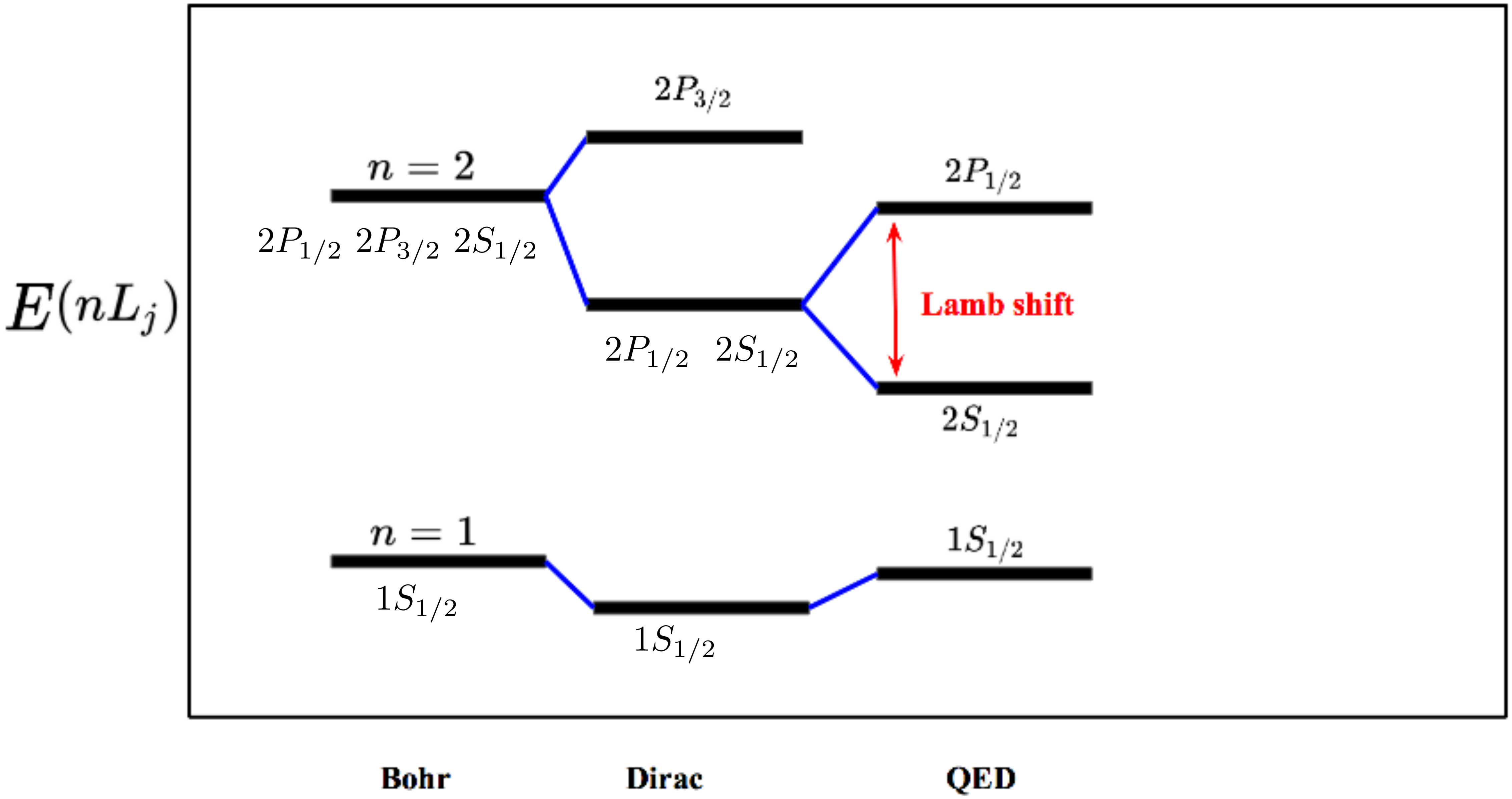

$E(nL_j)$
$2P_{3/2}$
$n = 2$
$2P_{1/2}\ 2P_{3/2}\ 2S_{1/2}$
$2P_{1/2}\quad 2S_{1/2}$
$n = 1$
$1S_{1/2}$
$1S_{1/2}$
Bohr
Dirac

Lamb Shift


$E(nL_j)$
$n = 2$
$2P_{1/2}$ $2P_{3/2}$ $2S_{1/2}$
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Lamb shift
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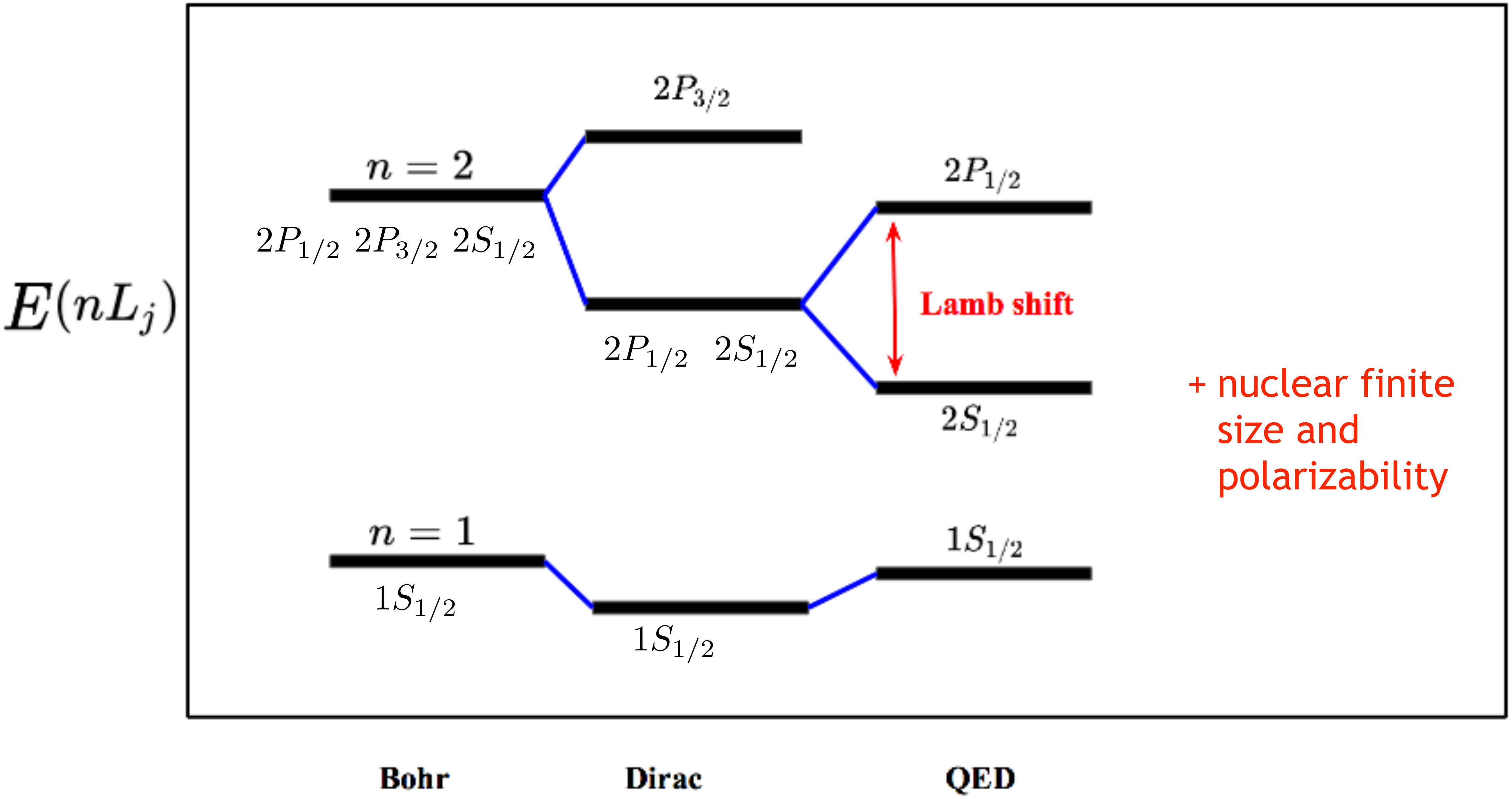
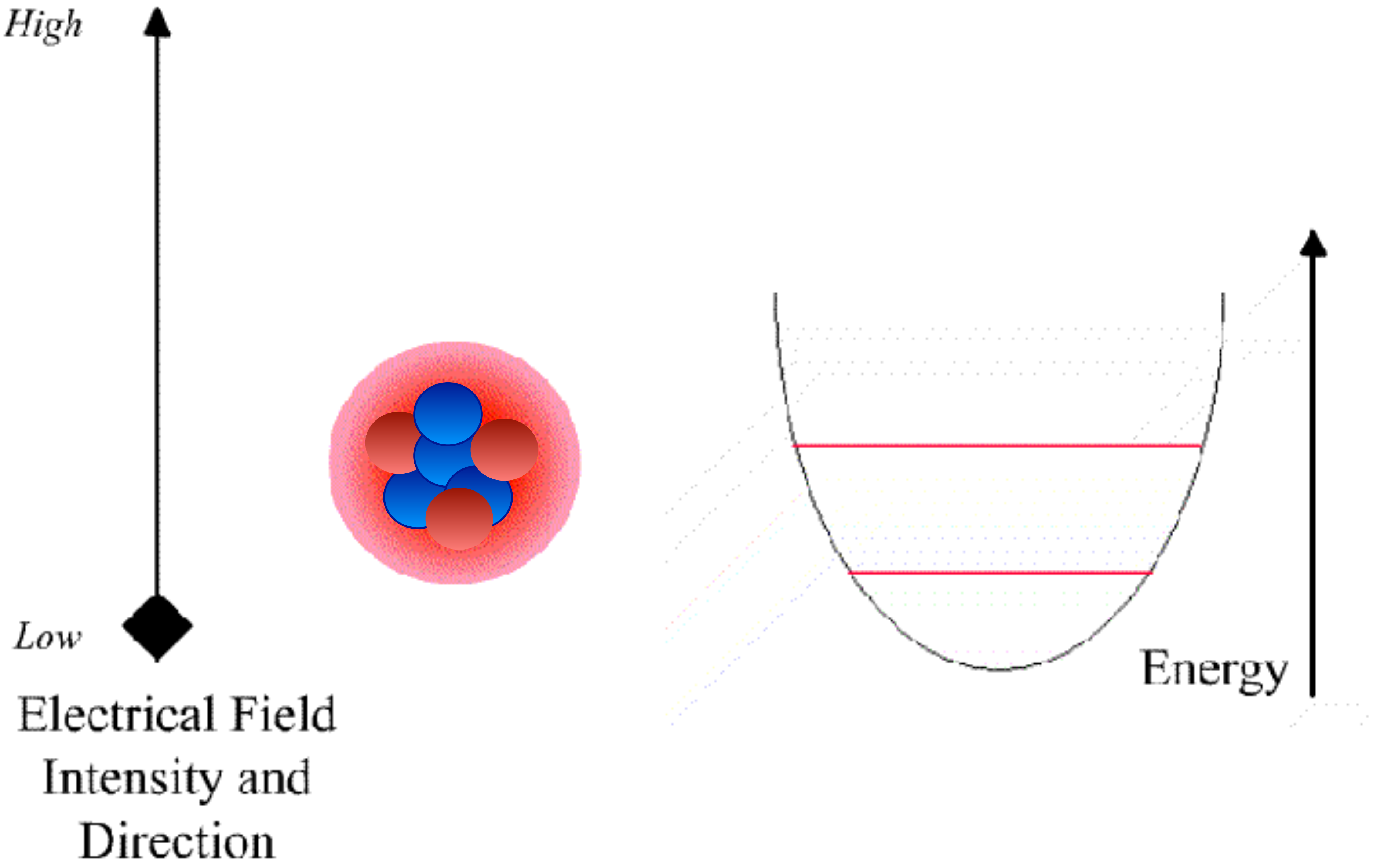


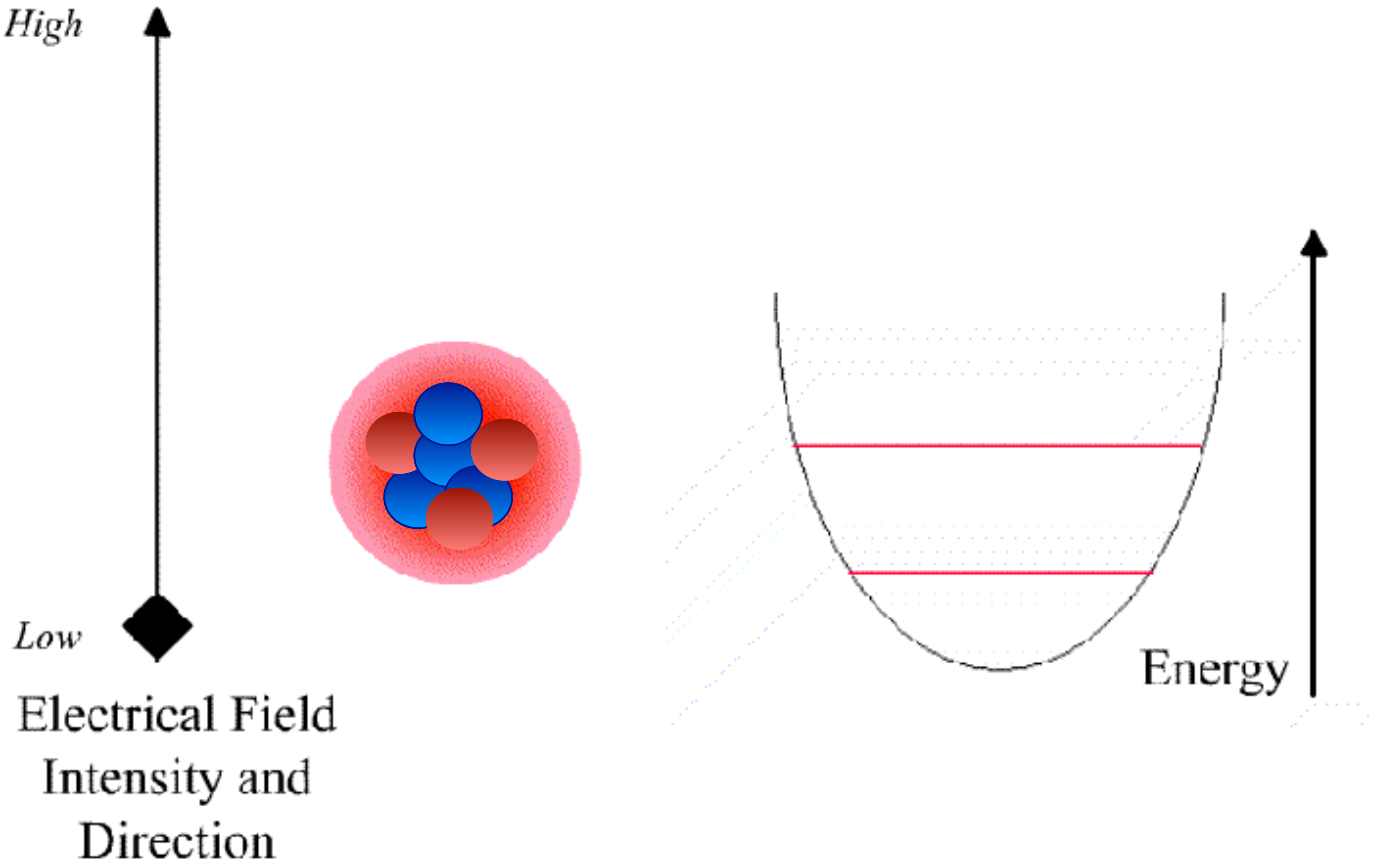
+ nuclear finite size and polarizability

Concept of polarizability



$$\mathbf{P} = \alpha_{\mathbf{E1}} \mathbf{E}$$

Concept of polarizability



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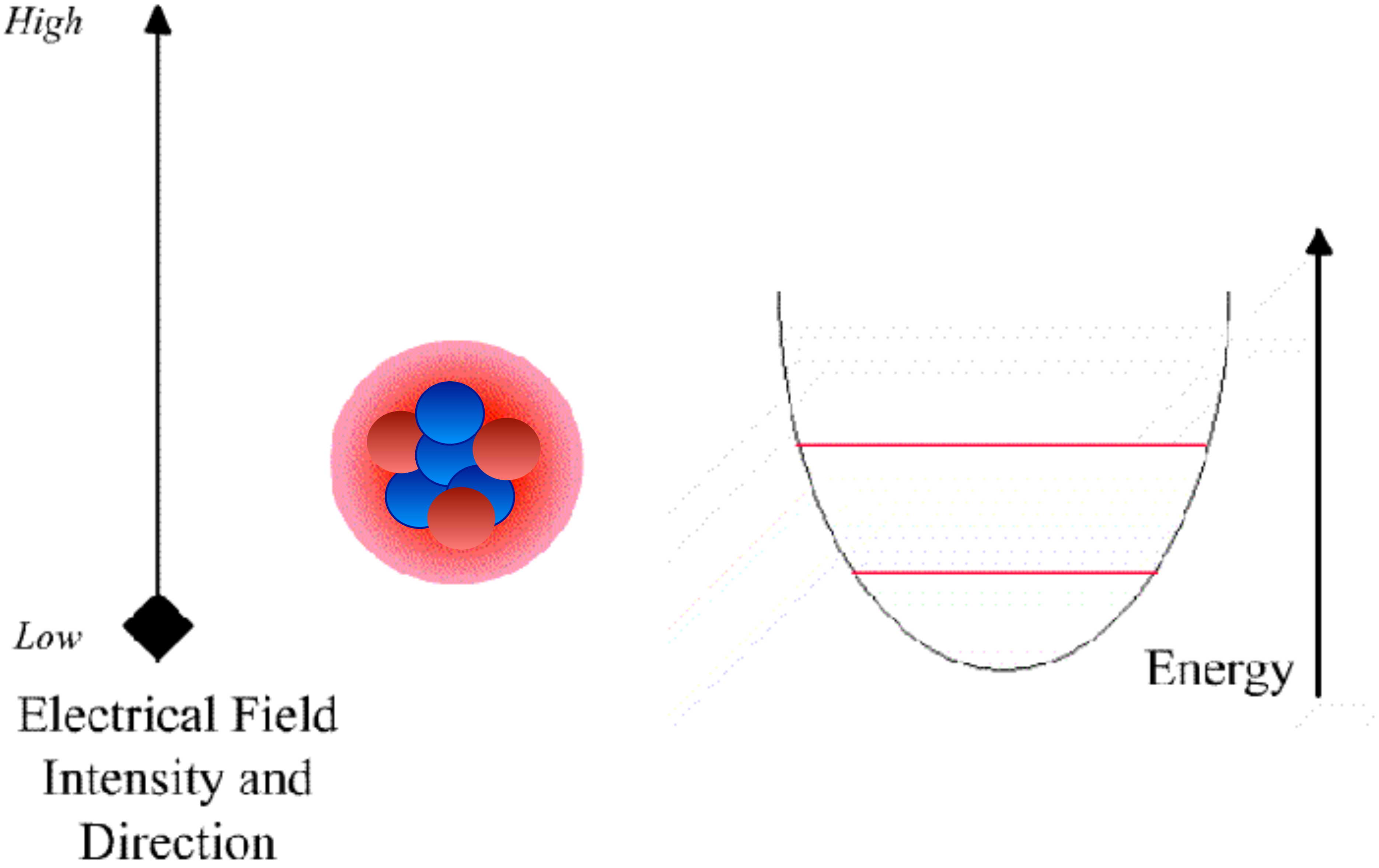

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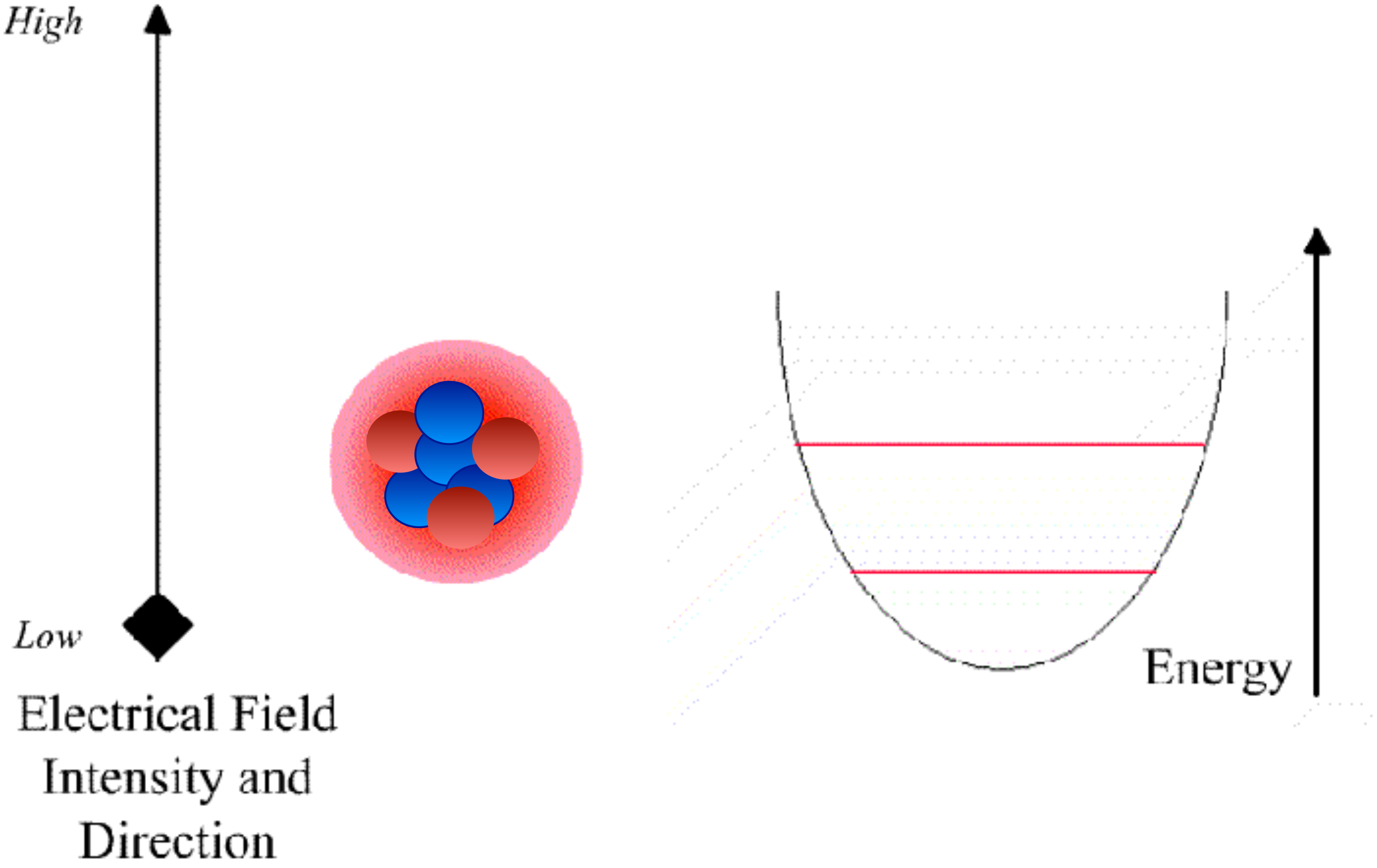

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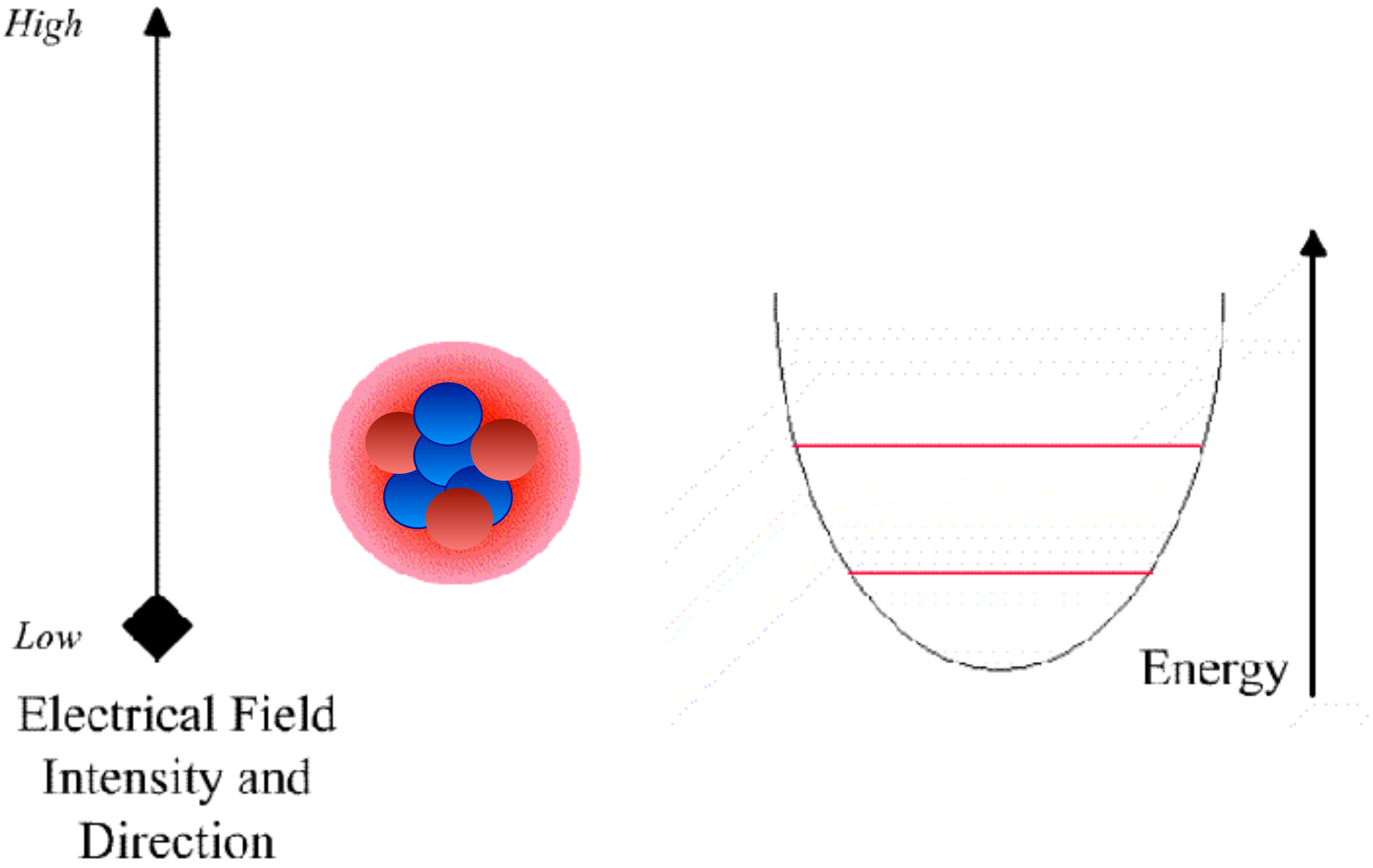

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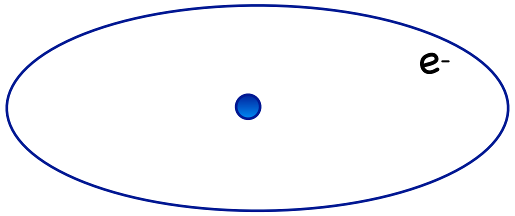
Proton Radius Puzzle

The proton charge radius is measured from:



- **electron-proton interactions**: 0.8770 ± 0.0045 fm

  eH spectroscopy

  e-p scattering

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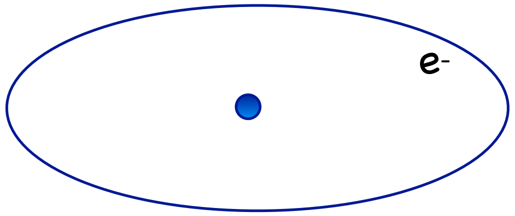
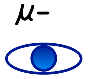


- **muonic-proton interactions**: 0.8409 ± 0.0004 fm
  - μH **Lamb-shift**



Pohl *et al.*, Nature (2010)
Antognini *et al.*, Science (2013)

nature

SHRINKING THE PROTON

New value for charge radius of tiny subatomic particle

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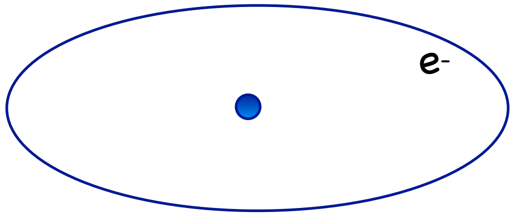

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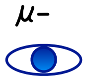

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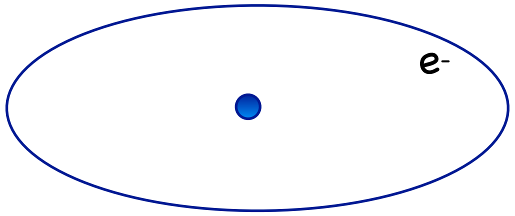

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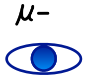

Pohl *et al.*, Nature (2010)
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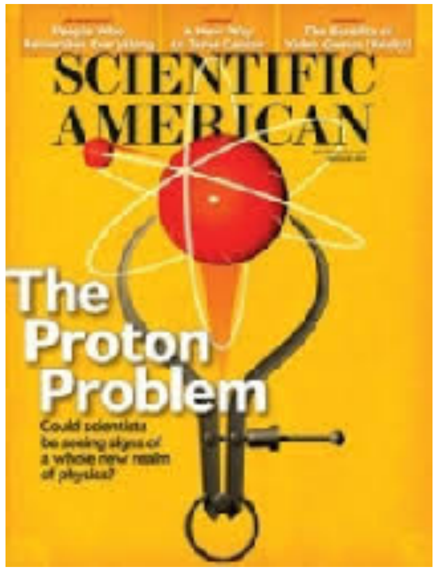

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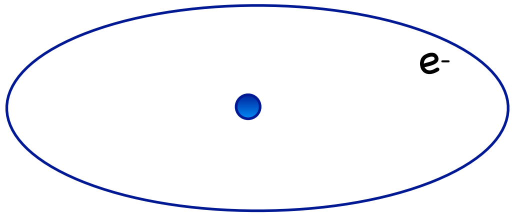

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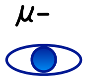

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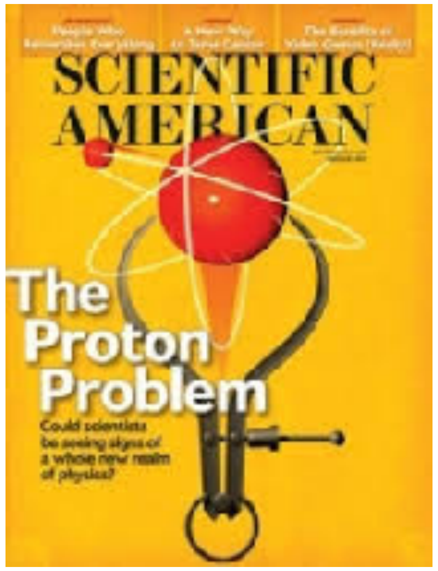

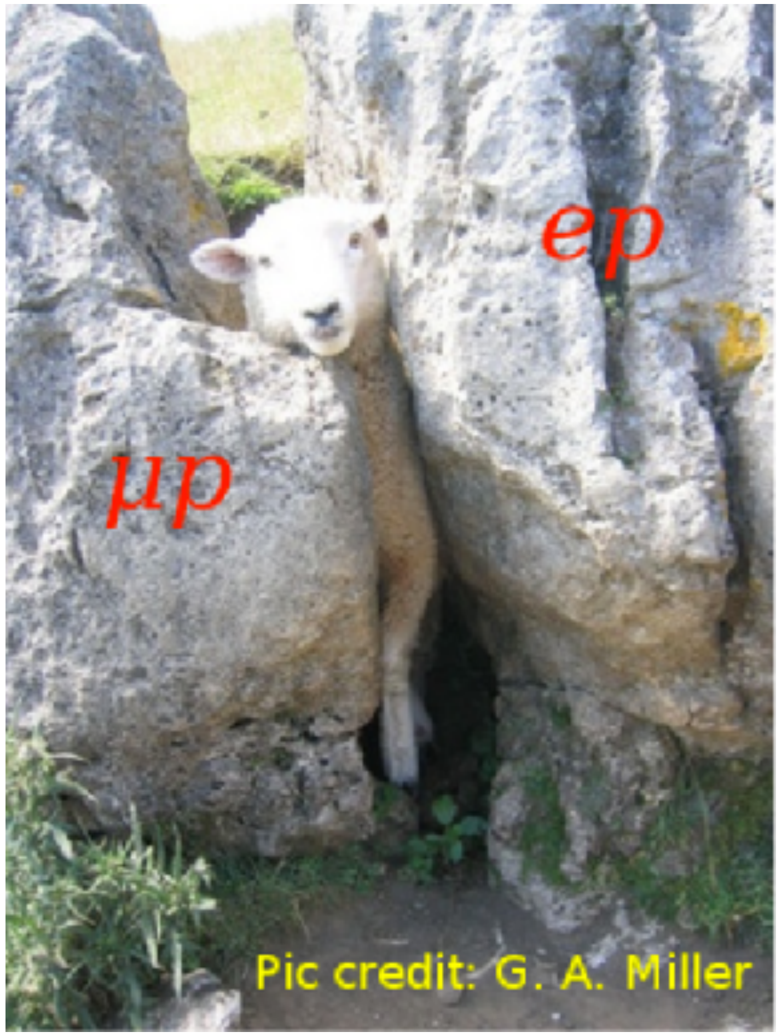

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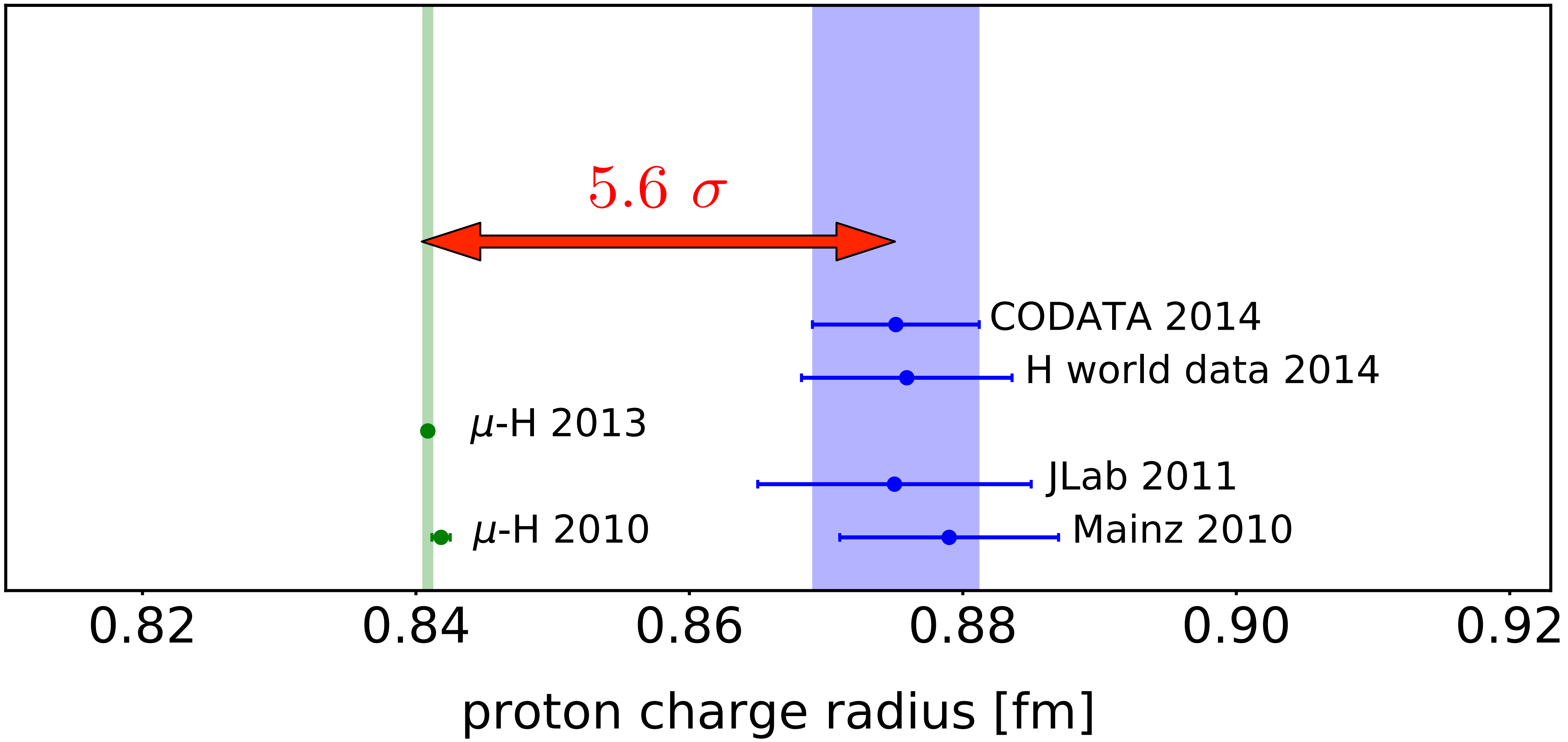
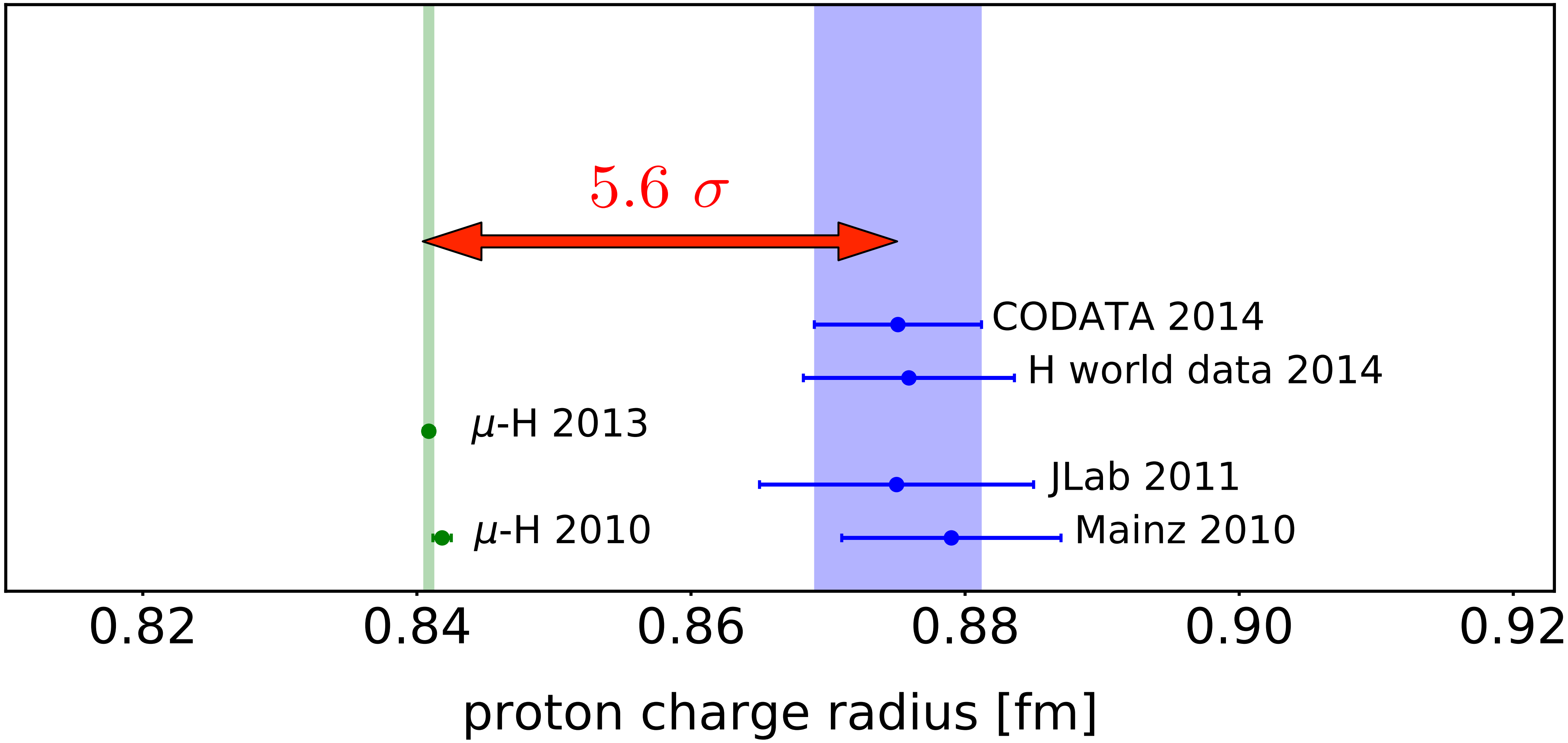


Proton Radius Puzzle

Is lepton universality violated?



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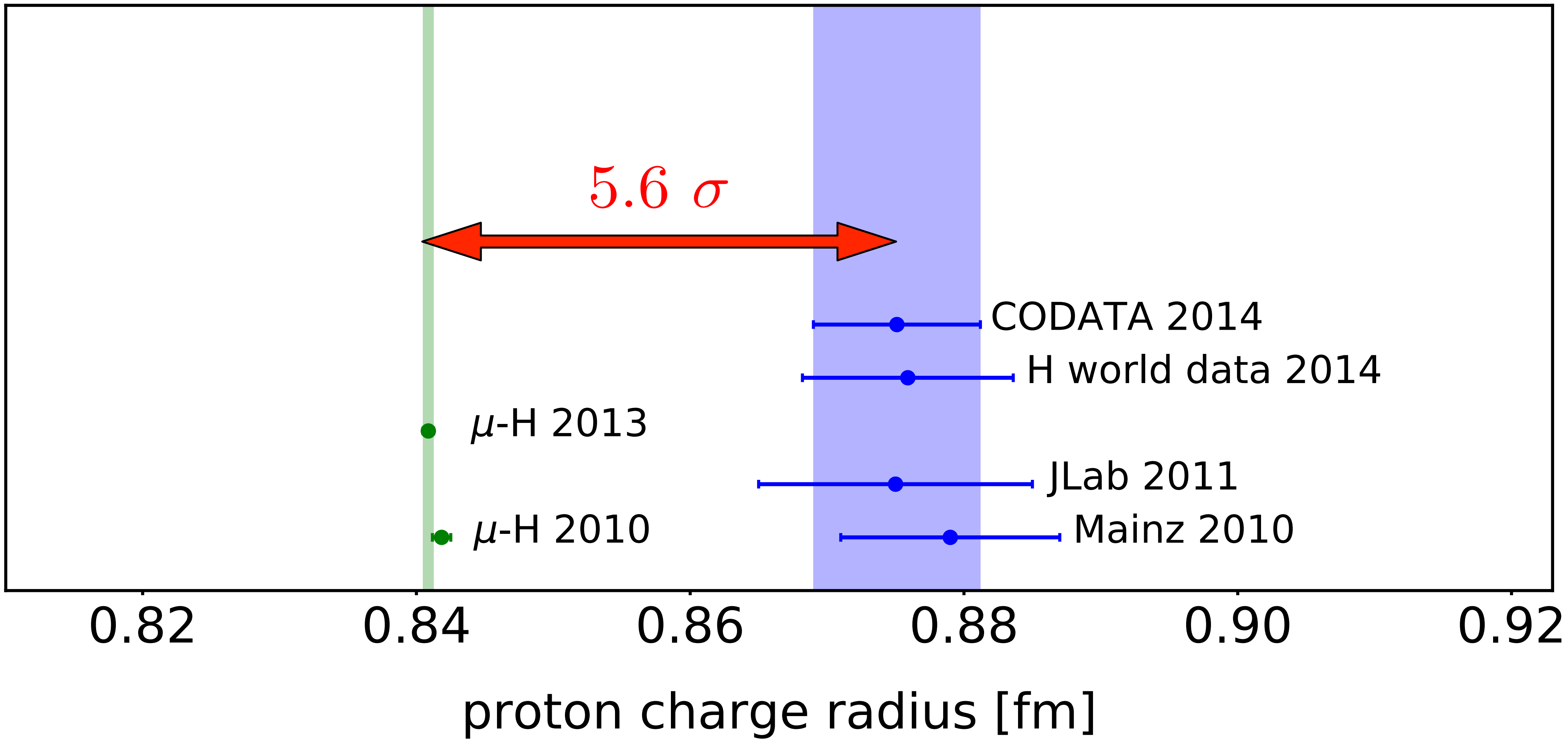
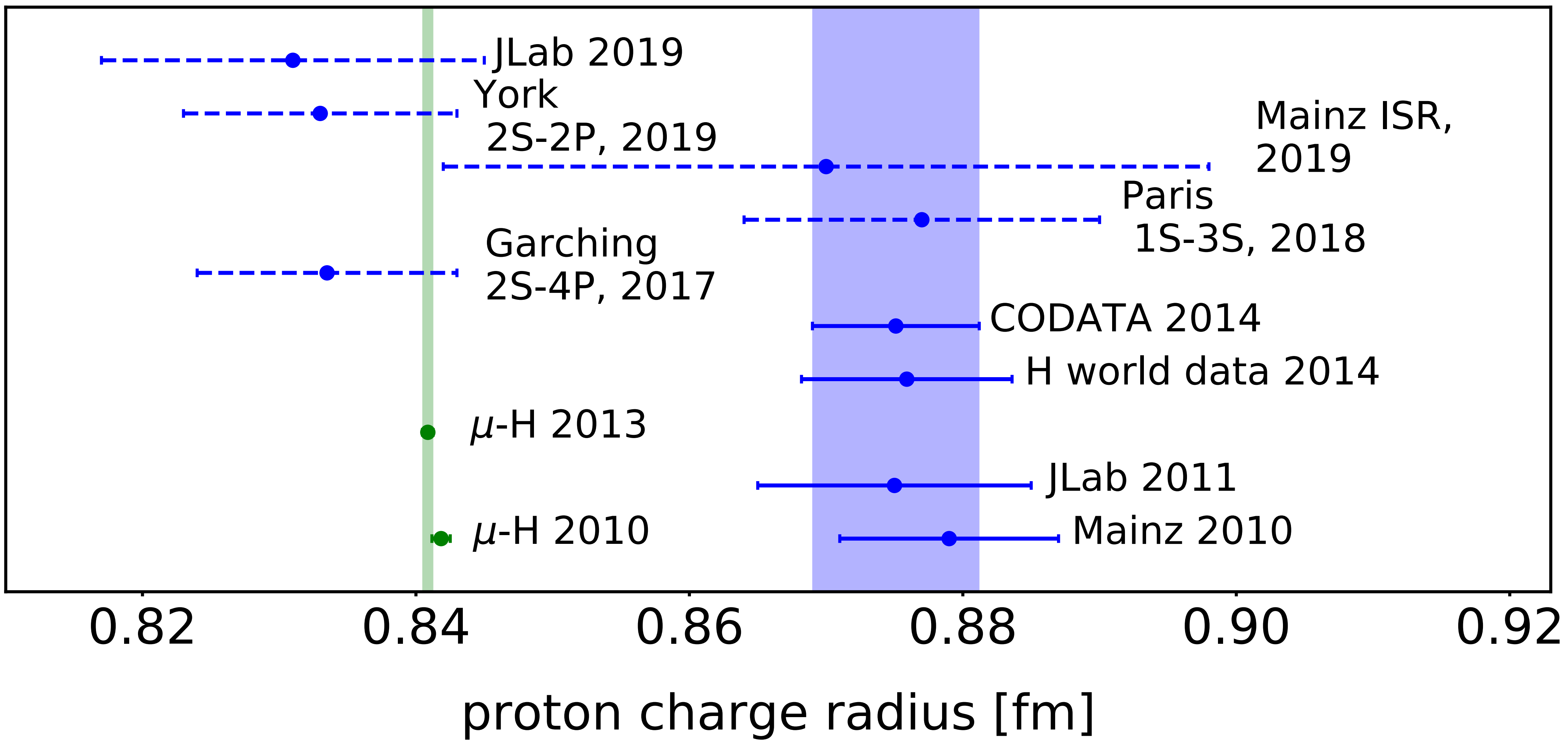


Possible beyond standard model explanations:

Batell, McKeen, Pospelov, PRL (2011)
Tucker-Smith, Yavin, PRD (2011)
Carlson Rislow, PRD (2014)
…

Proton Radius Puzzle

— Today's situation —



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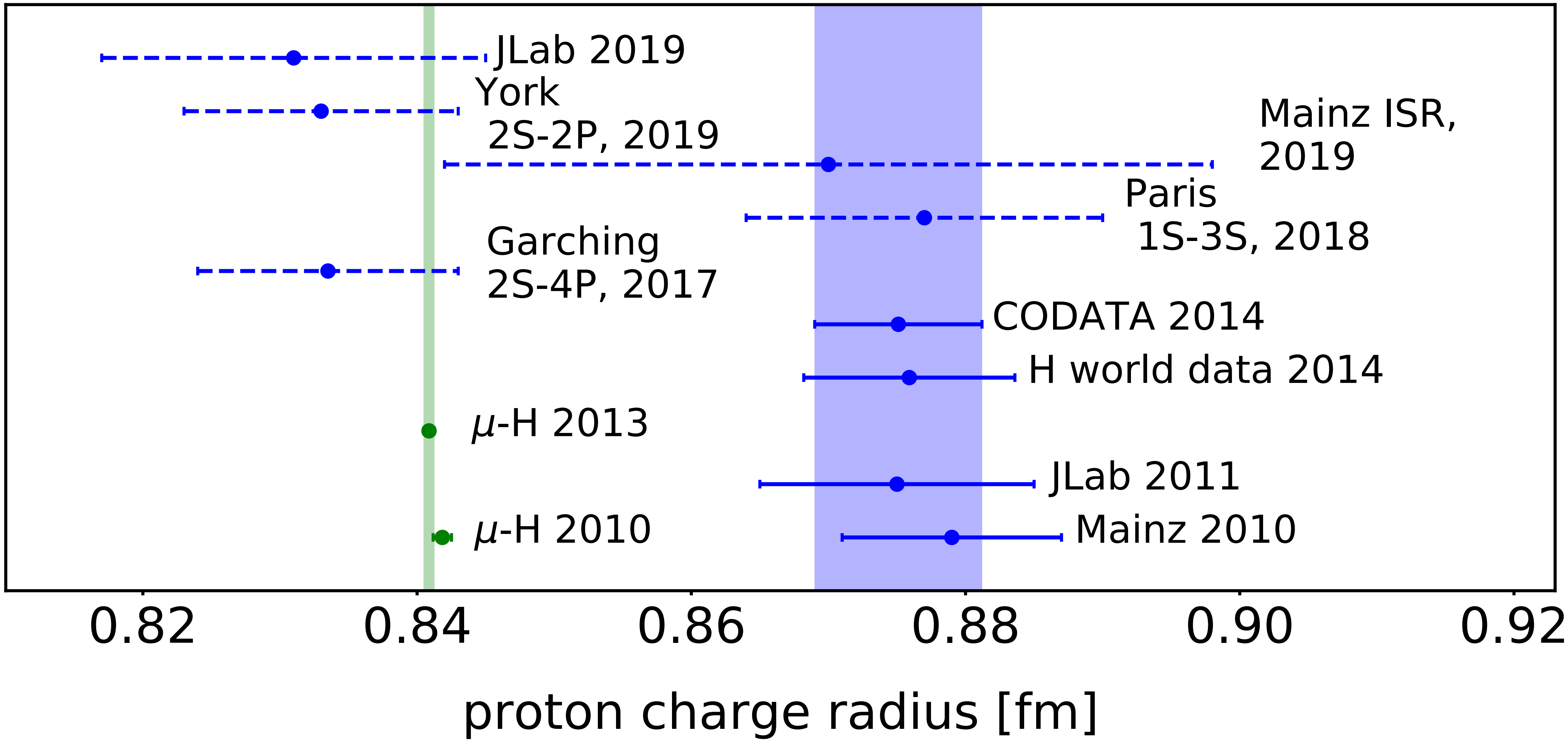
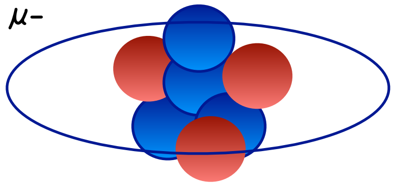


JLab 2019 Xiong et al., Nature **575**, 147-150 (2019)

York 2019 Berzginov et al., Science **365**, 1007-1012 (2019)

Understanding the Proton Radius Puzzle

Strong experimental program at PSI (Switzerland) from the CREMA collaboration to unravel the mystery by studying the **Lamb shift in other muonic atoms than μH:**



$$\Delta E_{2S-2P} = \delta_{\rm QED} + \frac{m_r^4 (Z\alpha)^4}{12} \langle r_c^2 \rangle + \delta_{\rm TPE}$$

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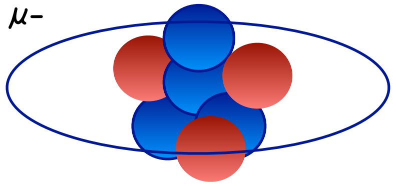

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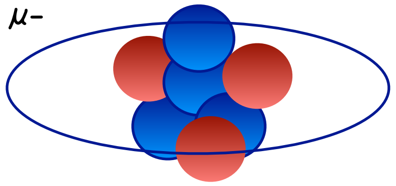

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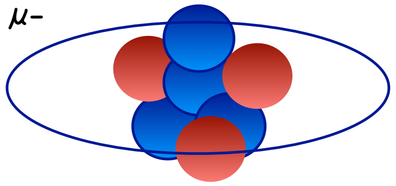

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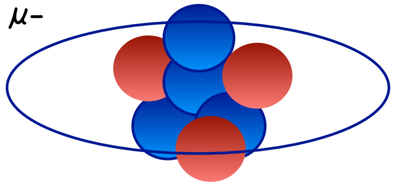

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well known

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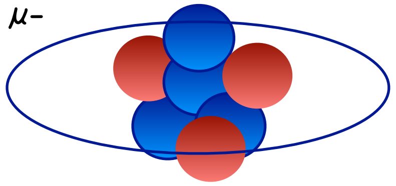

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not well known

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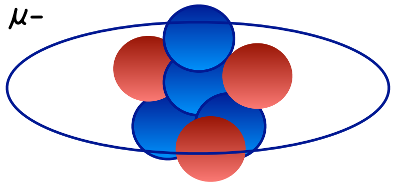

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well known

not well known

- μD $\longrightarrow$ results released in 2016
- μ^4He$^+$ $\longrightarrow$ analyzing data
- μ^3He$^+$ $\longrightarrow$ analyzing data
- μ^3H $\longrightarrow$ impossible because triton is radioactive
- μ^6Li^{2+} $\longrightarrow$ future plan
- μ^7Li^{2+} $\longrightarrow$ future plan

Nuclear structure corrections δ_{TPE}

TPE?

Nuclear structure corrections δ_{TPE}

TPE? ~~Airport code of Taiwan-Taoyuan~~

Nuclear structure corrections δ_{TPE}

TPE?

~~Airport code of Taiwan-Taoyuan~~

~~Tavola periodica degli elementi~~

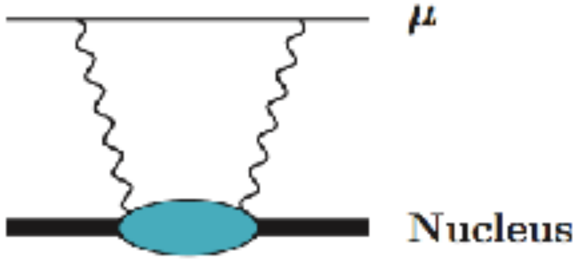
Nuclear structure corrections δ_{TPE}

TPE?

~~Airport code of Taiwan-Taoyuan~~

~~Tavola periodica degli elementi~~

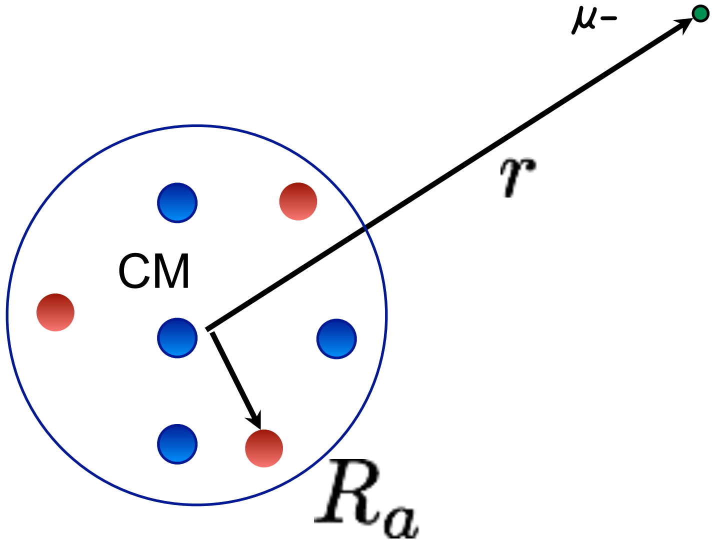
Two photon exchange diagram



Theoretical derivation of TPE

$$H = H_N + H_\mu + \Delta V$$

$$H_\mu = \frac{p^2}{2m_r} - \frac{Z\alpha}{r}$$



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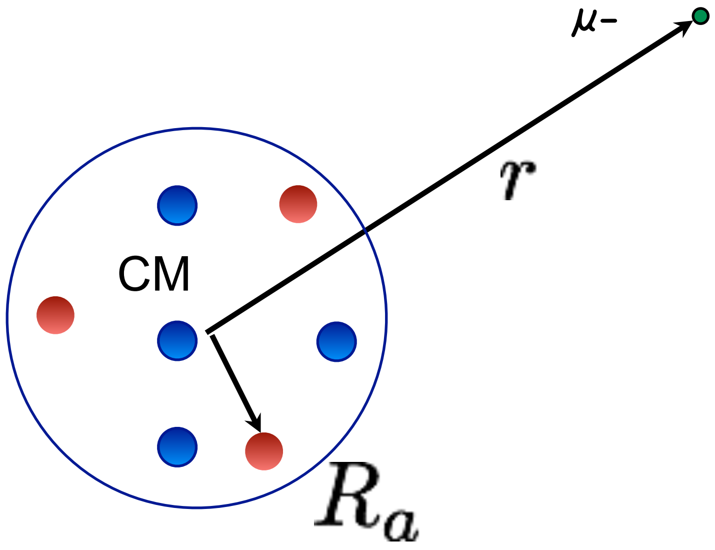

Perturbative potential: correction to the bulk Coulomb

$$\Delta V = \sum_a^Z \alpha \left(\frac{1}{r} - \frac{1}{|\vec{r} - \vec{R}_a|} \right)$$

Theoretical derivation of TPE

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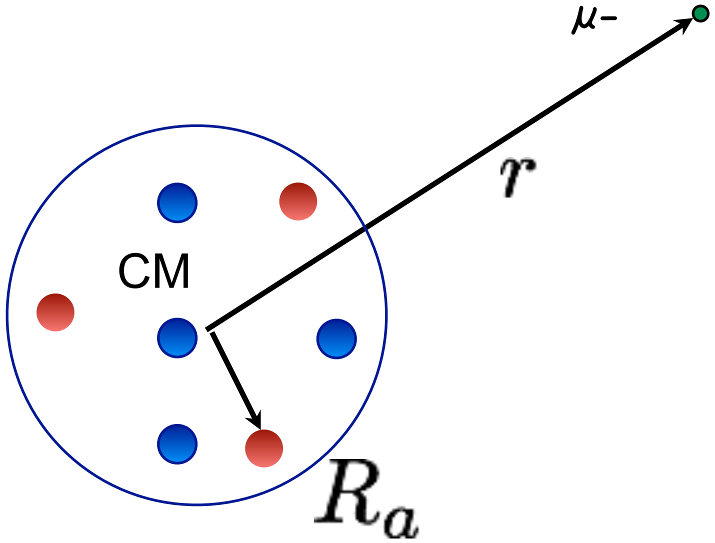
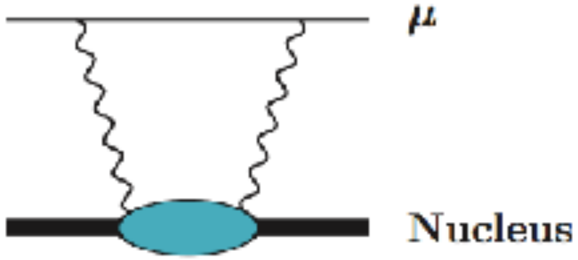


Perturbative potential: correction to the bulk Coulomb

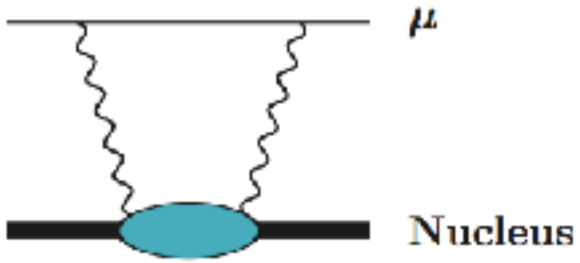
$$\Delta V = \sum_a^Z \alpha \left(\frac{1}{r} - \frac{1}{|\vec{r} - \vec{R}_a|} \right)$$

Using perturbation theory at second order one obtains the expression for TPE up to order $(Z\alpha)^5$



Theoretical derivation of TPE

- Non relativistic term



Take non-relativistic kinetic energy in muon propagator
Neglect Coulomb force in the intermediate state
Expand the muon matrix elements in powers of η

η= $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$

$$P \simeq \frac{m_r^3(Z\alpha)^5}{12}\sqrt{\frac{2m_r}{\omega}}\left[|\boldsymbol{R}-\boldsymbol{R}'|^2 - \frac{\sqrt{2m_r\omega}}{4}|\boldsymbol{R}-\boldsymbol{R}'|^3 + \frac{m_r\omega}{10}|\boldsymbol{R}-\boldsymbol{R}'|^4\right]$$

Theoretical derivation of TPE

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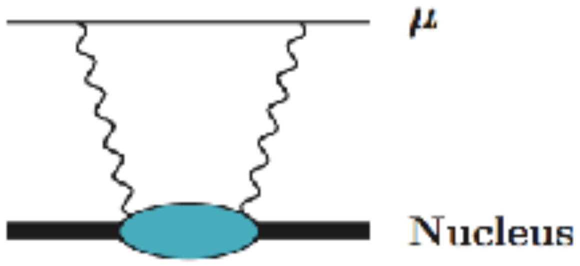
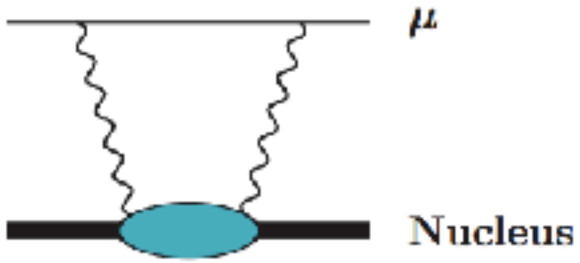


$$P \simeq \frac{m_r^3(Z\alpha)^5}{12}\sqrt{\frac{2m_r}{\omega}}\left[\underset{\delta^{(0)}}{\underset{\uparrow}{|\boldsymbol{R}-\boldsymbol{R}'|^2}} - \frac{\sqrt{2m_r\omega}}{4}\underset{\delta^{(1)}}{\underset{\uparrow}{|\boldsymbol{R}-\boldsymbol{R}'|^3}} + \frac{m_r\omega}{10}\underset{\delta^{(2)}}{\underset{\nearrow}{|\boldsymbol{R}-\boldsymbol{R}'|^4}}\right]$$

Theoretical derivation of TPE

Non relativistic term



Take non-relativistic kinetic energy in muon propagator
Neglect Coulomb force in the intermediate state
Expand the muon matrix elements in powers of η

η= $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$

$$P \simeq \frac{m_r^3(Z\alpha)^5}{12}\sqrt{\frac{2m_r}{\omega}}\Big[\underbrace{|\boldsymbol{R}-\boldsymbol{R}'|^2}_{\delta^{(0)}} - \frac{\sqrt{2m_r\omega}}{4}\underbrace{|\boldsymbol{R}-\boldsymbol{R}'|^3}_{\delta^{(1)}} + \frac{m_r\omega}{10}\underbrace{|\boldsymbol{R}-\boldsymbol{R}'|^4}_{\delta^{(2)}}\Big]$$

- ★ $|\boldsymbol{R}-\boldsymbol{R}'|$ “virtual” distance traveled by the proton between the two-photon exchange
- ★ Uncertainty principle $|\boldsymbol{R}-\boldsymbol{R}'| \sim \frac{1}{\sqrt{2m_N\omega}}$
- ★ $\eta = \sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{\frac{m_r}{m_N}} = 0.33$

Theoretical derivation of TPE

- Non relativistic term

★ $\delta^{(0)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^2$

dominant term, related to the energy-weighted integral

$$\delta^{(0)}_{D1} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S_{D_1}(\omega)$$

of the dipole response function

$$S_{D1}(\omega) = \frac{1}{2J_0+1} \sum\!\!\!\!\!\!\int_{N \neq N_0} |\langle NJ||\hat{D}_1||N_0 J_0\rangle|^2 \delta(\omega - \omega_N)$$

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★ $\delta^{(1)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^3$ Related to Zemach moment elastic contribution

$$\delta^{(1)}_{Z3} = \frac{\pi}{3} m_r (Z\alpha)^2 \phi^2(0) \iint d^3R d^3R' |\boldsymbol{R} - \boldsymbol{R}'|^3 \, \rho_0^p(\boldsymbol{R}) \rho_0^p(\boldsymbol{R}')$$

Theoretical derivation of TPE

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★ $\delta^{(2)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^4$

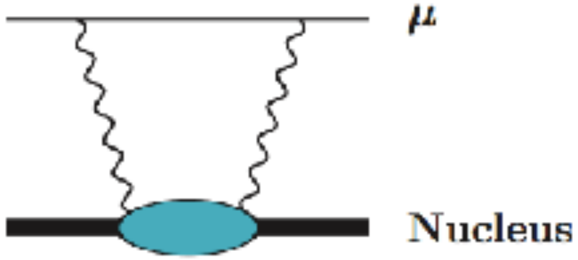
leads to energy-weighted integrals of three different response functions

$S_{R^2}(\omega), S_Q(\omega), S_{D1D3}(\omega)$

Theoretical derivation of TPE

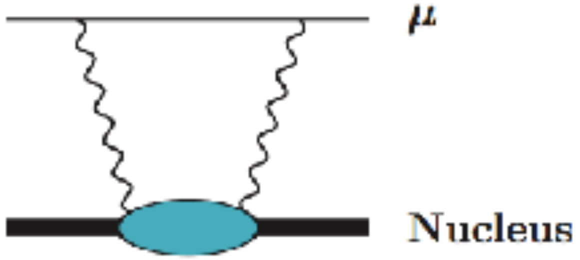
- Coulomb term

Consider the Coulomb force in the intermediate states
Naively $\delta_C^{(0)} \sim (Z\alpha)^6$, actually logarithmically enhanced
$\delta_C^{(0)} \sim (Z\alpha)^5 \log(Z\alpha)$ Friar (1977), Pachucki (2011)
Related to the dipole response function



Theoretical derivation of TPE

Coulomb term



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Relativistic terms

Take the relativistic kinetic energy in muon propagator
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$$\delta_{L(T)}^{(0)} = \frac{2m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega\, K_{L(T)}\left(\frac{\omega}{m_r}\right)\; S_{D_1}(\omega)$$

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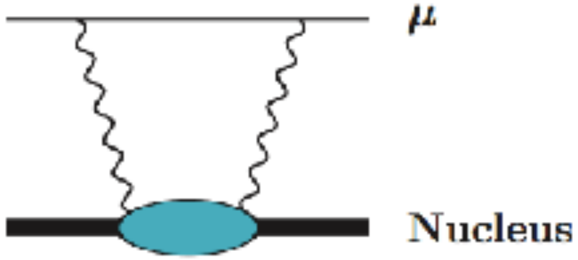

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- Finite nucleon-size corrections

Consider finite nucleon-size by including their charge distributions and obtain terms, e.g.,

$$\delta_{R1}^{(1)} = -8\pi m_r (Z\alpha)^2 \phi^2(0) \int\int d^3R d^3R' |\boldsymbol{R} - \boldsymbol{R}'| \left[\frac{2}{\beta^2}\rho_0^{pp}(\boldsymbol{R},\boldsymbol{R}') - \lambda\rho_0^{np}(\boldsymbol{R},\boldsymbol{R}')\right]$$

Theoretical derivation of TPE

$$\delta_{\rm TPE} = \delta^{A}_{\rm Zem} + \delta^{N}_{\rm Zem} + \delta^{A}_{\rm pol} + \delta^{N}_{\rm pol}$$

Theoretical derivation of TPE

$$\delta_{\mathrm{TPE}} = \delta^{A}_{\mathrm{Zem}} + \delta^{N}_{\mathrm{Zem}} + \delta^{A}_{\mathrm{pol}} + \delta^{N}_{\mathrm{pol}}$$

$$\begin{aligned}\delta^{A}_{\mathrm{pol}} = {} & \delta^{(0)}_{D1} + \delta^{(1)}_{R3} + \delta^{(1)}_{Z3} + \delta^{(2)}_{R^2} + \delta^{(2)}_{Q} + \delta^{(2)}_{D1D3} + \delta^{(0)}_{C} \\ & + \delta^{(0)}_{L} + \delta^{(0)}_{T} + \delta^{(0)}_{M} + \delta^{(1)}_{R1} + \delta^{(1)}_{Z1} + \delta^{(2)}_{NS}\end{aligned}$$

Theoretical derivation of TPE

$$\delta_{\rm TPE} = \delta^A_{\rm Zem} + \delta^N_{\rm Zem} + \delta^A_{\rm pol} + \delta^N_{\rm pol}$$

$$\delta^A_{\rm pol} = \delta^{(0)}_{D1} + \delta^{(1)}_{R3} + \delta^{(1)}_{Z3} + \delta^{(2)}_{R^2} + \delta^{(2)}_{Q} + \delta^{(2)}_{D1D3} + \delta^{(0)}_{C}$$
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$$\delta^A_{\rm Zem} = -\delta^{(1)}_{Z3} - \delta^{(1)}_{Z1}$$

Theoretical derivation of TPE

$$\delta_{\rm TPE} = \delta^{A}_{\rm Zem} + \delta^{N}_{\rm Zem} + \delta^{A}_{\rm pol} + \delta^{N}_{\rm pol}$$

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$$\delta^{A}_{\rm Zem} = -\cancel{\delta^{(1)}_{Z3}} - \cancel{\delta^{(1)}_{Z1}}$$

Friar an Payne ('97)

A matter of precision

The uncertainty of the extracted radius depends on the precision of the TPE

$$\Delta E_{2S-2P} = \delta_{\rm QED} + \mathcal{A}_{\rm OPE}\langle r_c^2\rangle + \delta_{\rm TPE}$$

Even though, roughly: 95% 4% 1%

The uncertainty on TPE exceeds the experimental precision, hence reducing uncertainties is important

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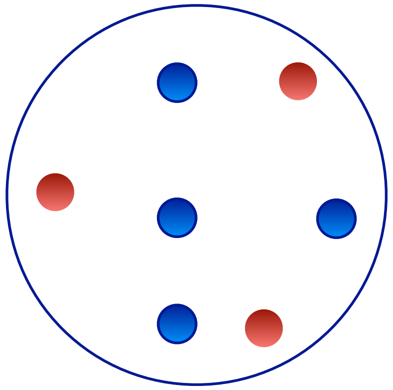
Uncertainties comparison

Atom	Exp uncertainty on $\Delta E_{2S\text{-}2P}$	Uncertainty on TPE prior to the discovery of the proton radius puzzle
μ^2H	0.003 meV	0.03 meV
μ^3He$^+$	0.08 meV	1 meV
μ^4He$^+$	0.06 meV	0.6 meV
$\mu\ ^{6,7}$Li^{++}	0.7 meV	4 meV

Ab Initio Nuclear Theory

- Start from nuclear Hamiltonians

$$H_N = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \dots$$



- Solve the Schrödinger equation for few-nucleons

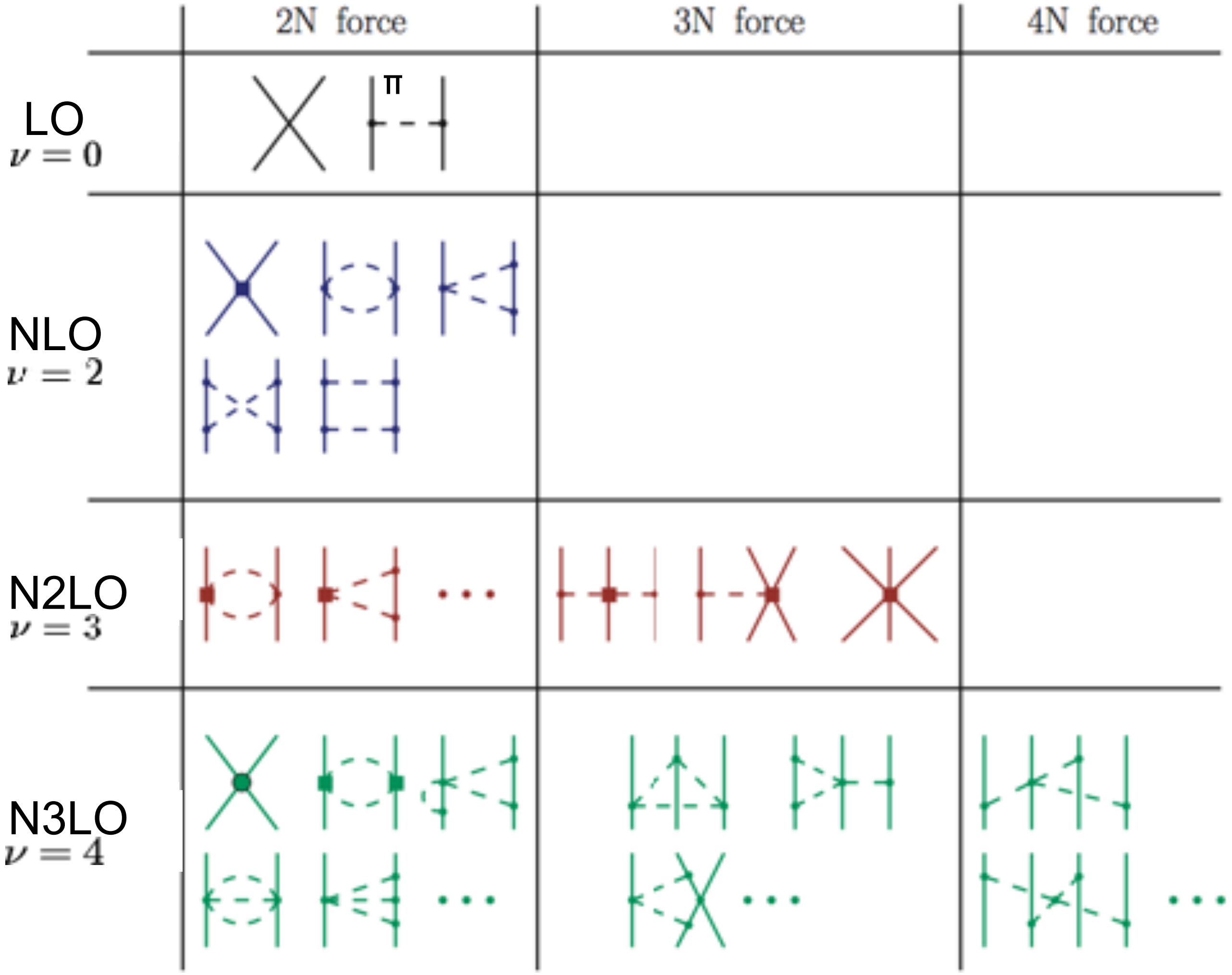
$$H_N|\psi_i\rangle = E_i|\psi_i\rangle$$

using numerical methods that allow to assign uncertainties

Nuclear Hamiltonians

- Chiral effective filed theory

Systematic expansion $\mathcal{L} = \sum_{\nu} c_{\nu} \left(\frac{Q}{\Lambda_b} \right)^{\nu}$



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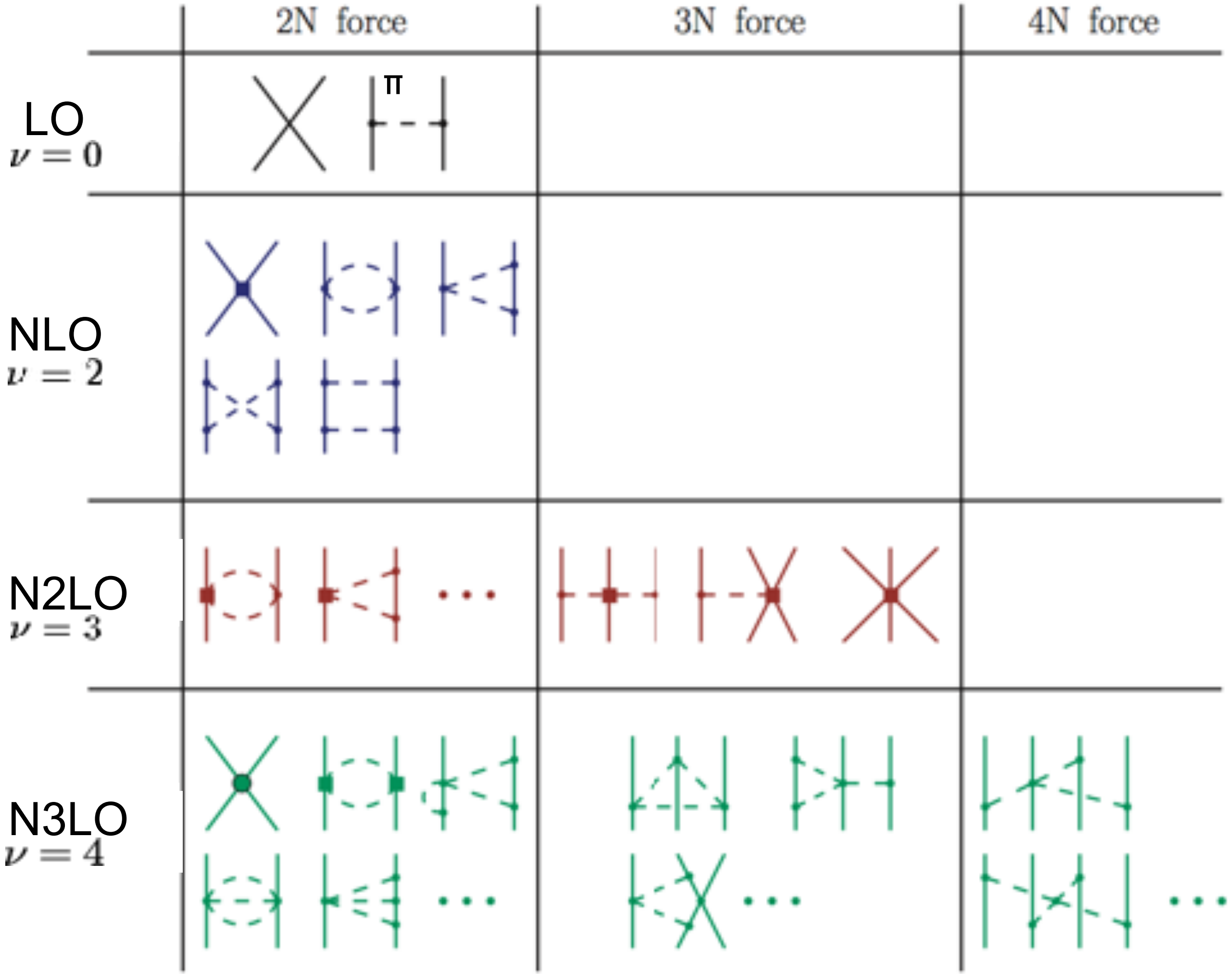
Details of short distance physics not resolved, but captured in low energy constants (LEC)



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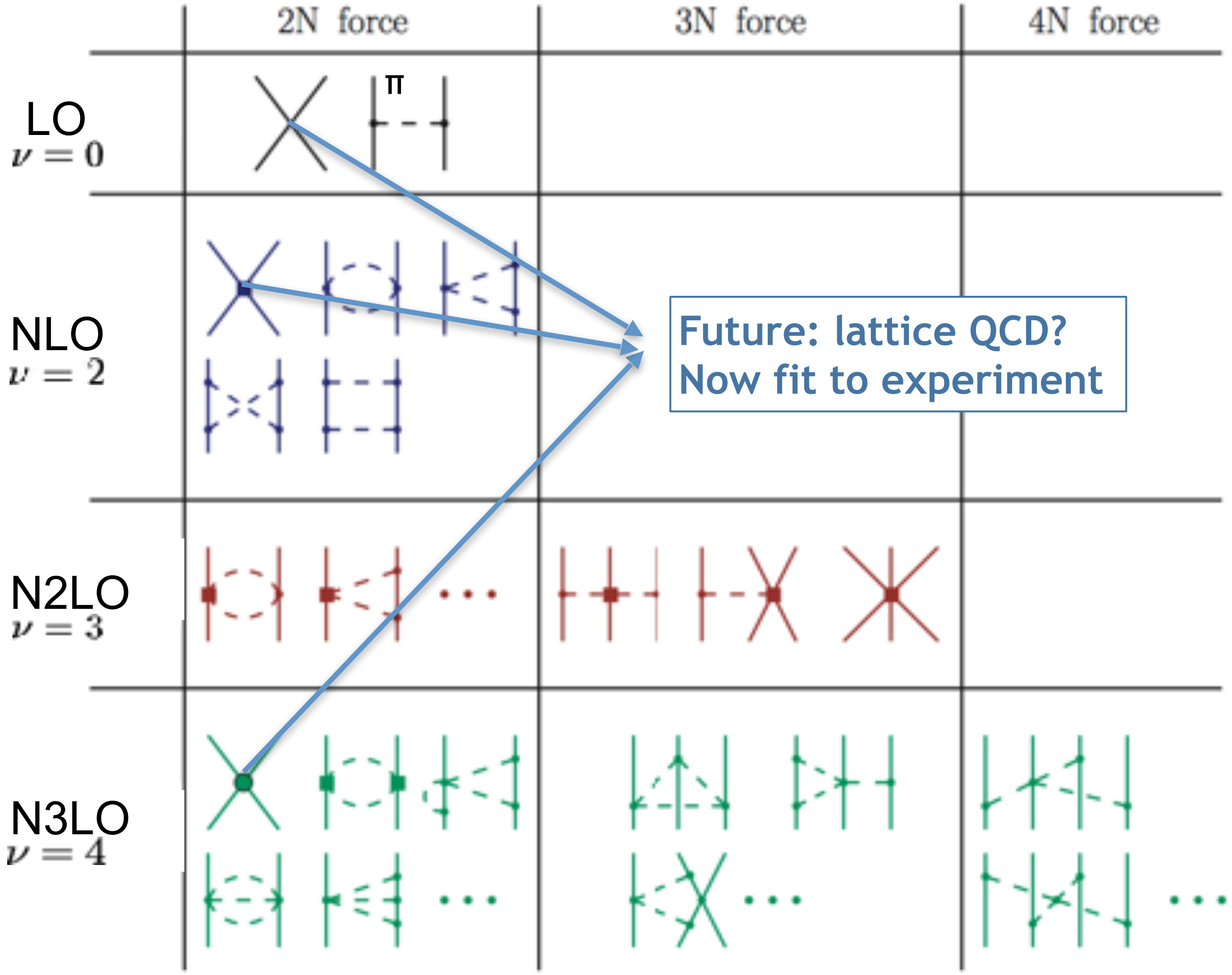

Future: lattice QCD?
Now fit to experiment

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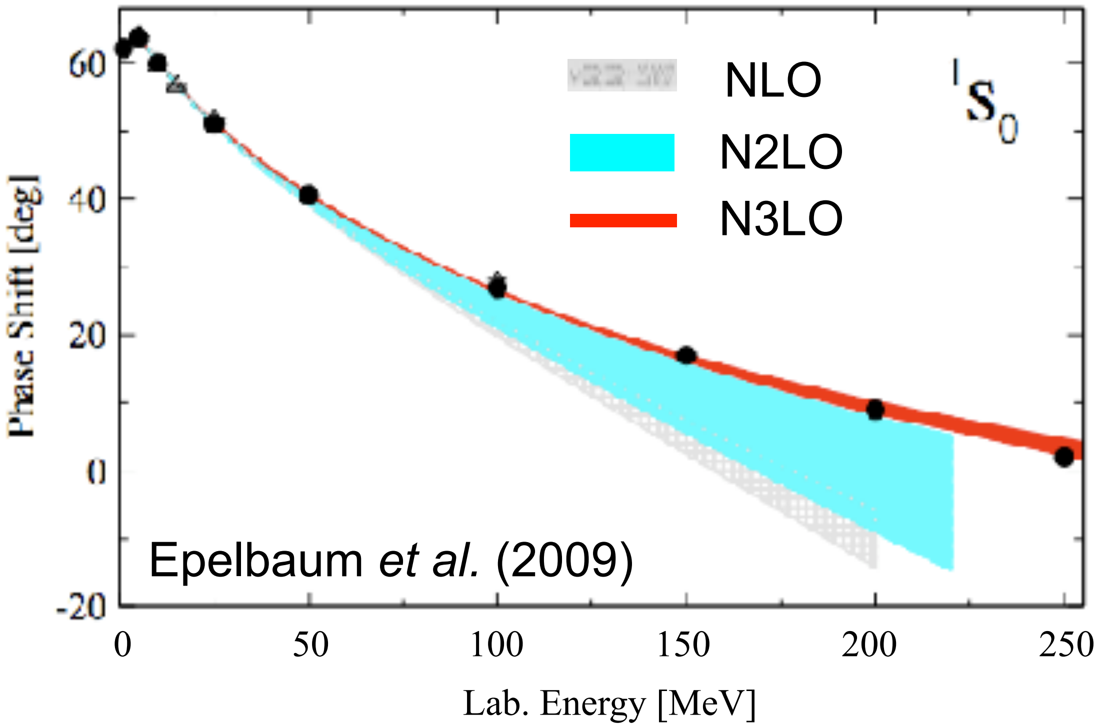
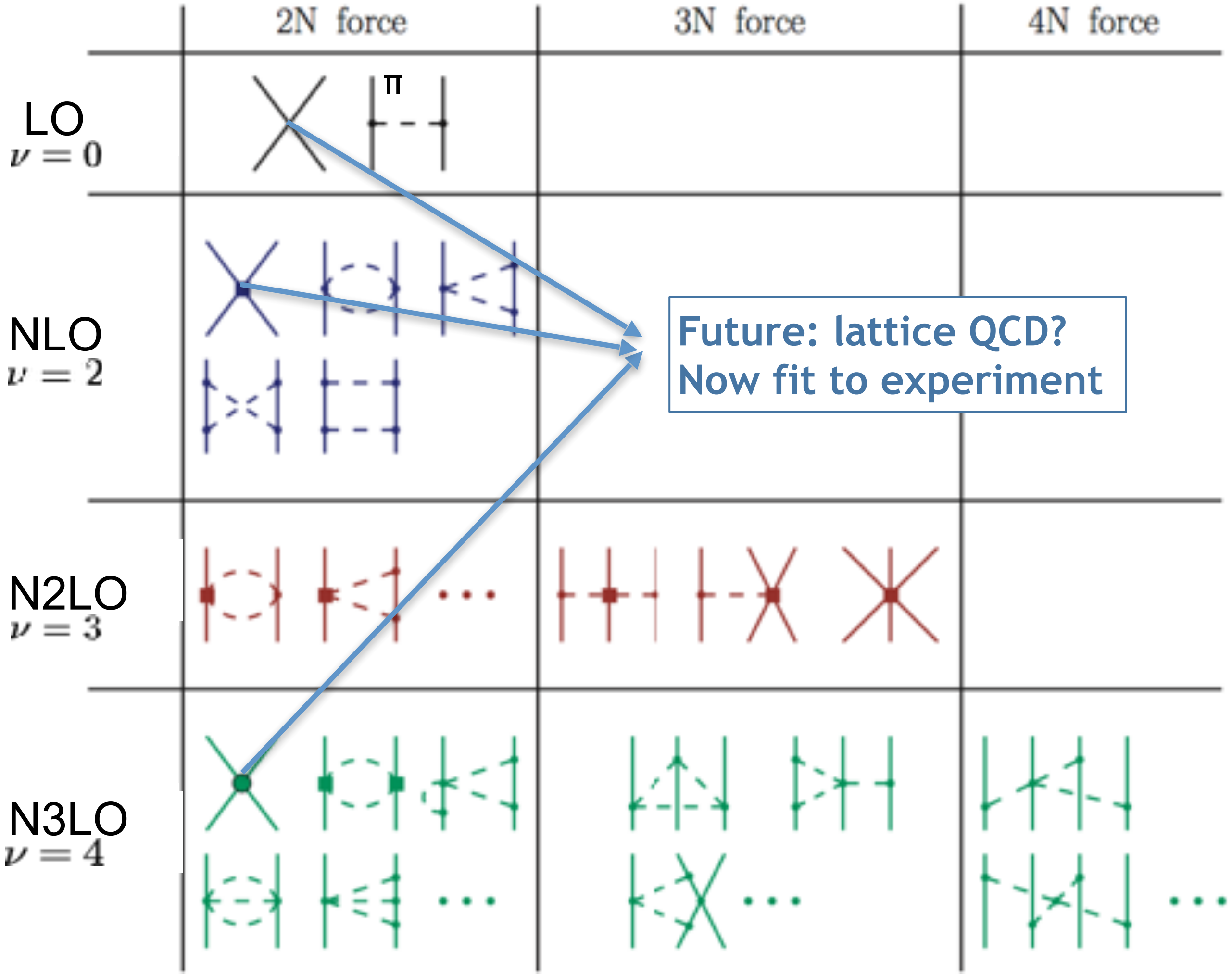
LEC fit to experiment - NN sector -



Epelbaum *et al.* (2009)



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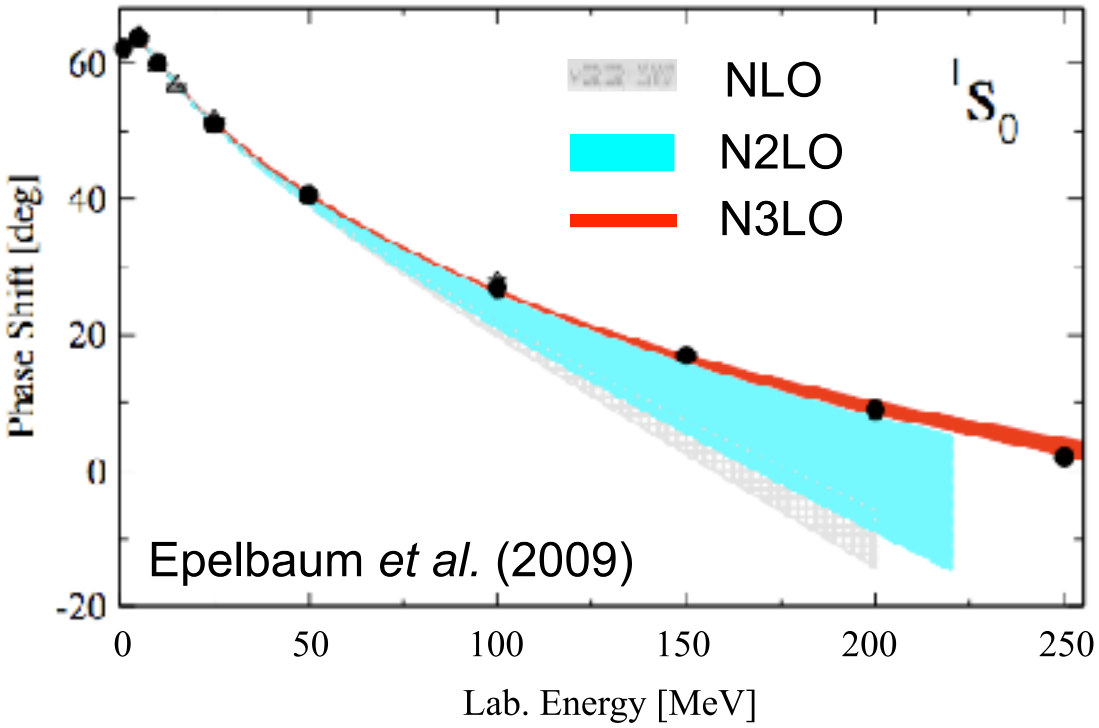
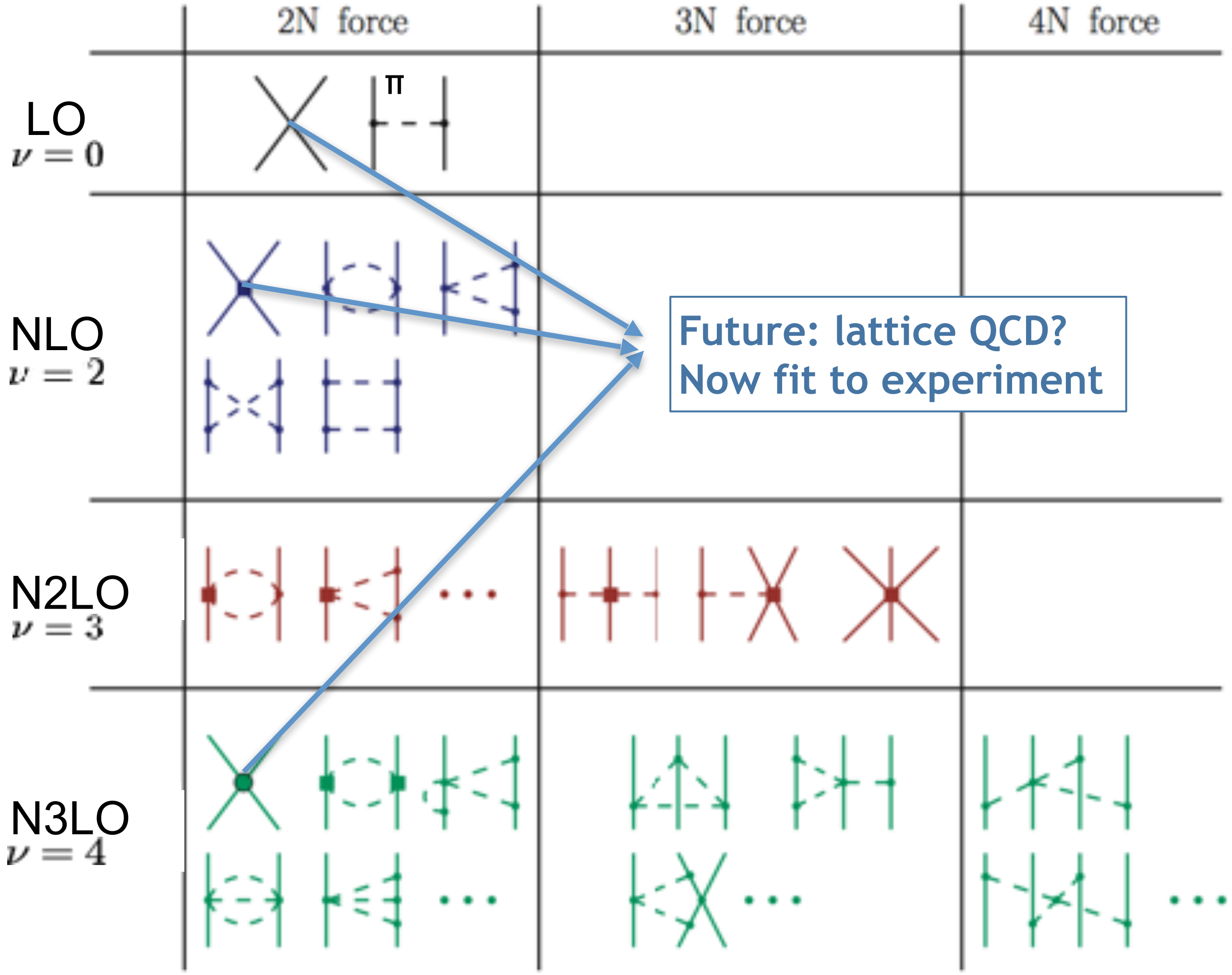




Future: lattice QCD?
Now fit to experiment

- Traditional hamiltonians

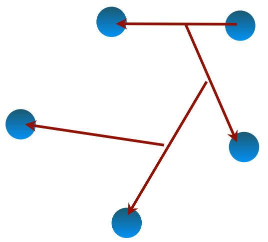
Exploit all other symmetries (e.g. translational, rotational invariance) but the chiral; use some ansatz for short range physics;
Fit NN phase shifts

Hyperspherical Harmonics

A from 3 up to 6, 7

(for A=2 we use an harmonic oscillator basis)

$$|\psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1,\vec{\eta}_2,\ldots,\vec{\eta}_{A-1})\rangle$$



Recursive definition of hyper-spherical coordinates

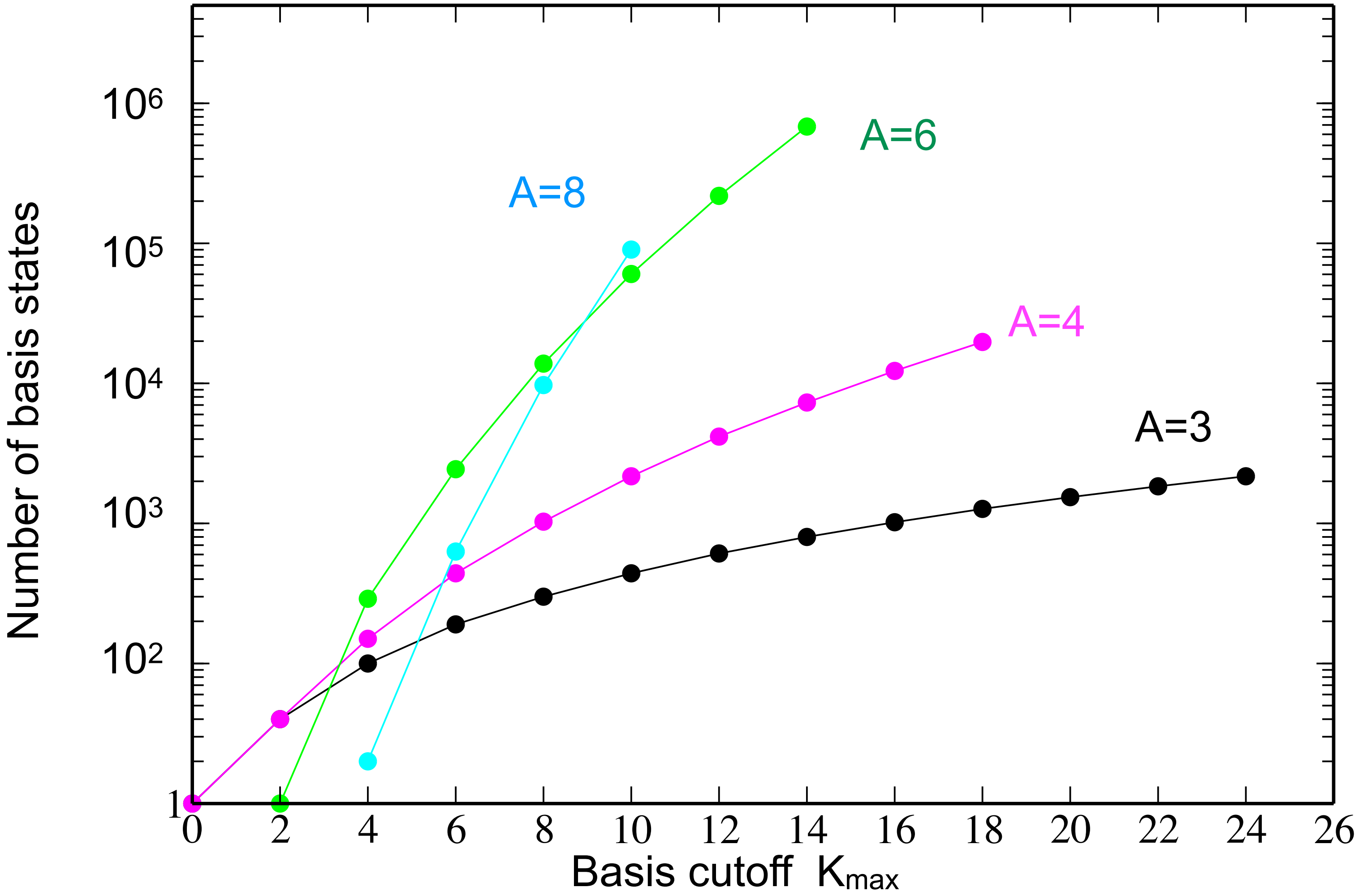
$\vec{\eta}_0 = \sqrt{A}\vec{R}_{CM}$ $\quad \vec{\eta}_1, ..., \vec{\eta}_{A-1}$

ρ, Ω $\quad \rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$

$$\Psi = \sum_{[K],\nu}^{K_{max},\nu_{max}} c_\nu^{[K]} e^{-\rho/2} \rho^{n/2} L_\nu^n\left(\frac{\rho}{b}\right) [\mathcal{Y}^{\mu}_{[K]}(\Omega) \chi^{\bar{\mu}}_{ST}]^a_{JT}$$

Hyperspherical Harmonics

A from 3 up to 6, 7

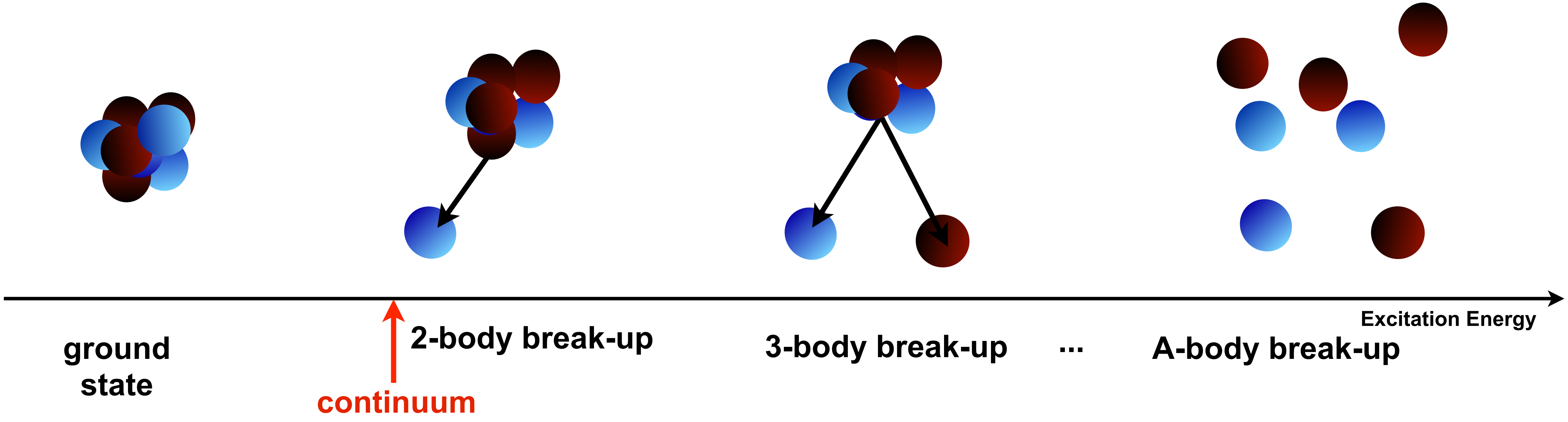


Exact method

Bad computational scaling

Lorentz integral transform method

Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459

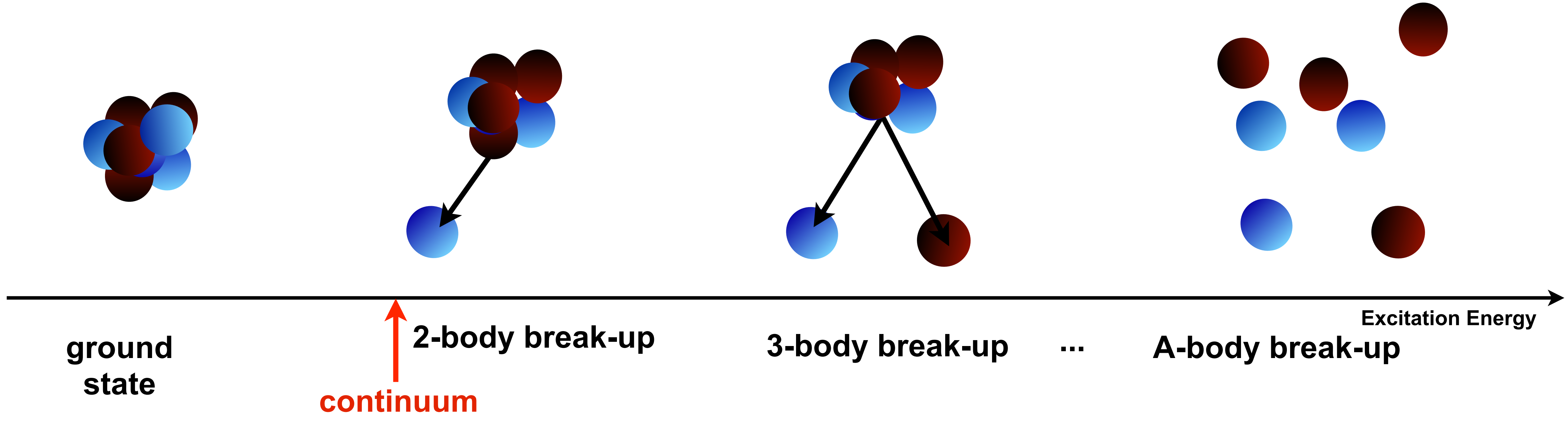


$$S(\omega) \rightarrow |\langle NJ||\hat{O}||N_0 J_0\rangle|^2$$

Exact knowledge limited in energy and mass number

Lorentz integral transform method

Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459



Excitation Energy

ground state

continuum

2-body break-up

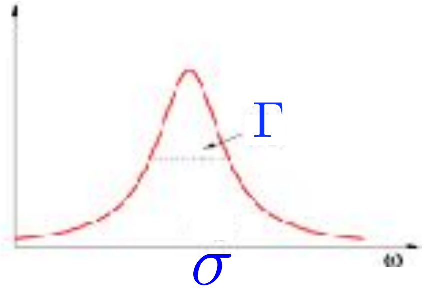
3-body break-up

...

A-body break-up

$$S(\omega) \to |\langle NJ||\hat{O}||N_0 J_0\rangle|^2 \quad \Longleftrightarrow \quad L(\sigma,\Gamma) = \frac{\Gamma}{\pi}\int d\omega \frac{S(\omega)}{(\omega-\sigma)^2+\Gamma^2} = \langle\tilde{\psi}|\tilde{\psi}\rangle$$

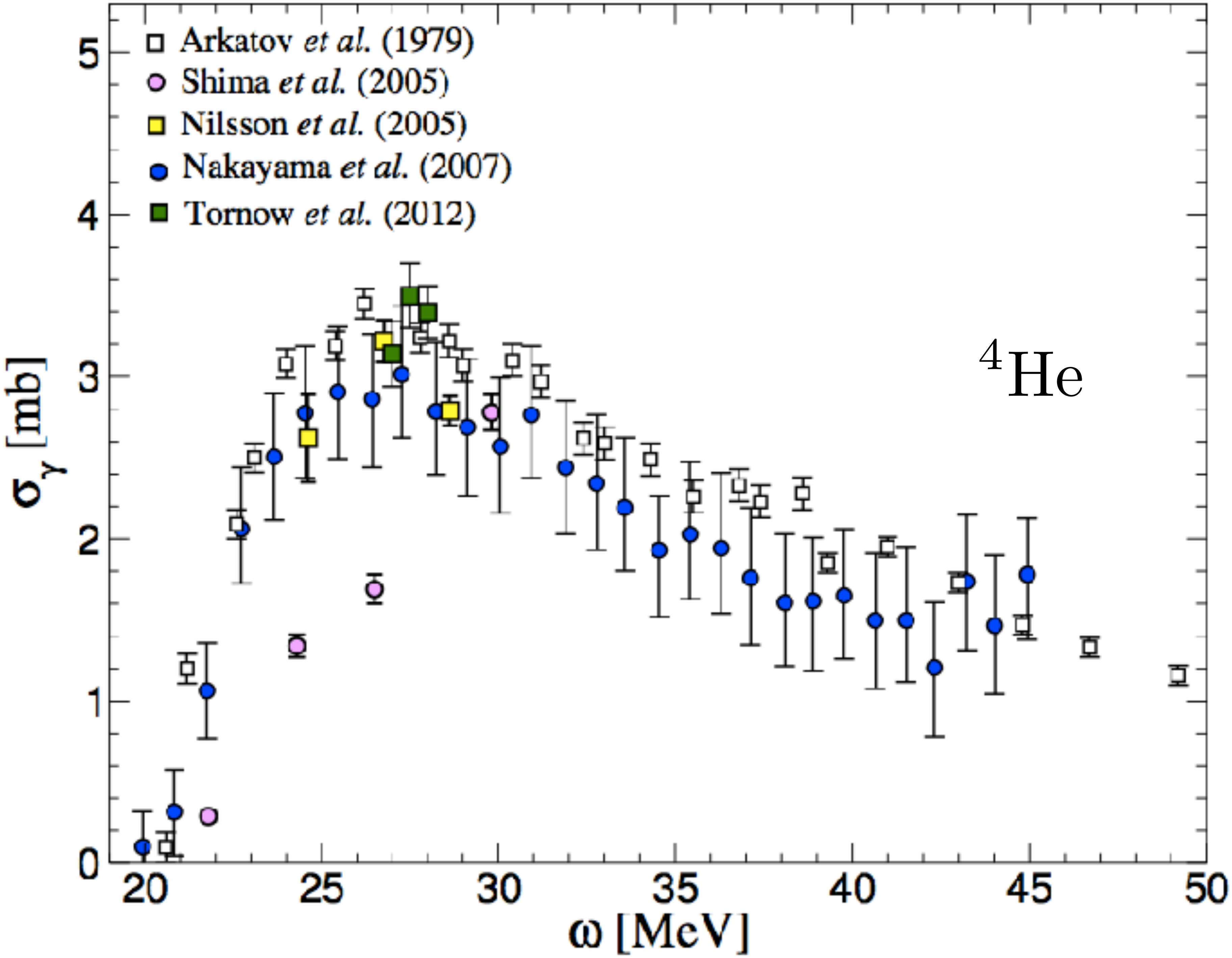
Exact knowledge limited in energy and mass number



$$\boxed{(H - E_0 - \sigma + i\Gamma)\,|\,\tilde{\psi}\rangle = \Theta\,|\,\psi_0\rangle}$$

Reduce the continuum problem to a bound-state-like equation

An example



S.B. and Saori Pastore, Journal of Physics G.: Nucl. Part. Phys. **41**, 123002 (2014)

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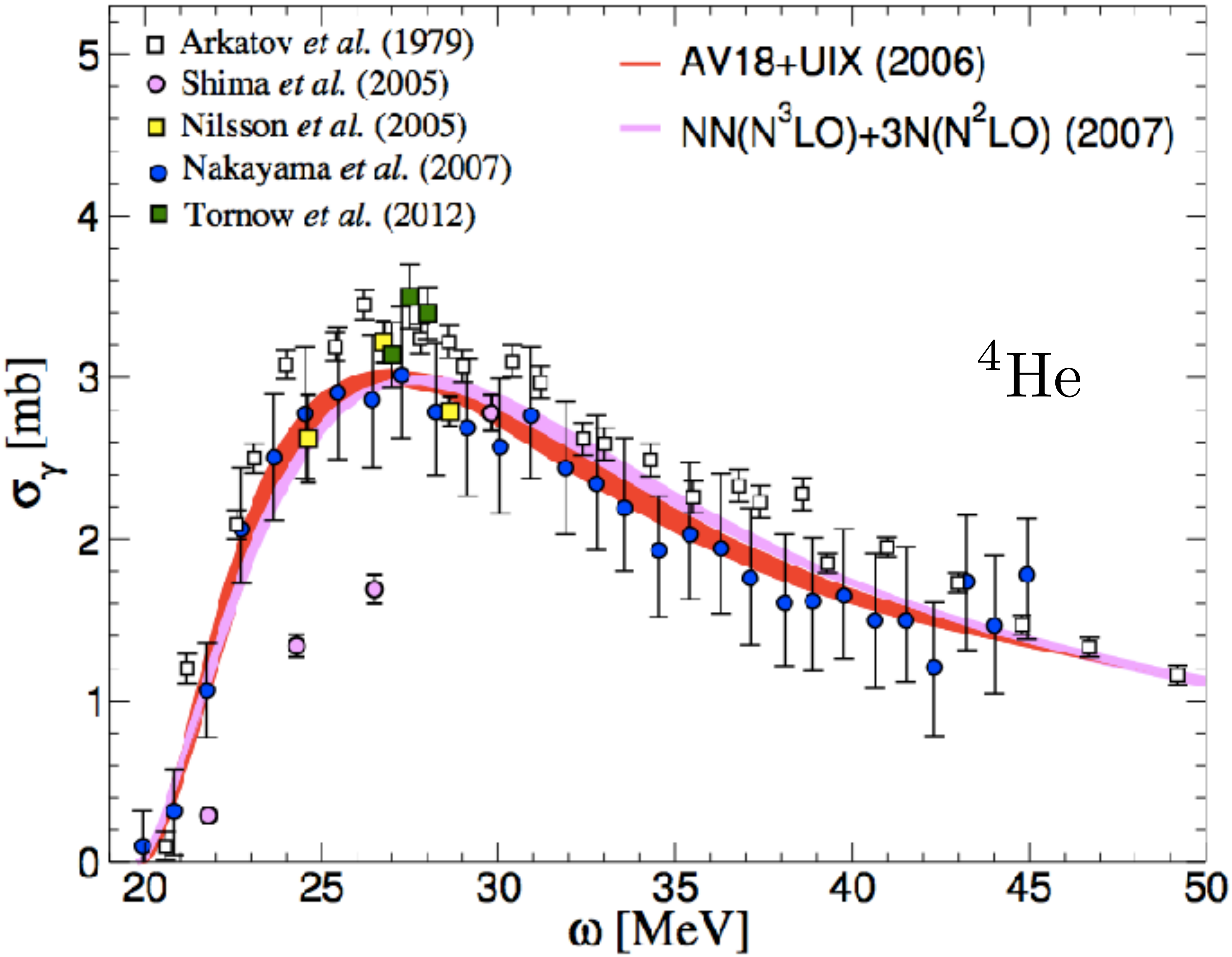

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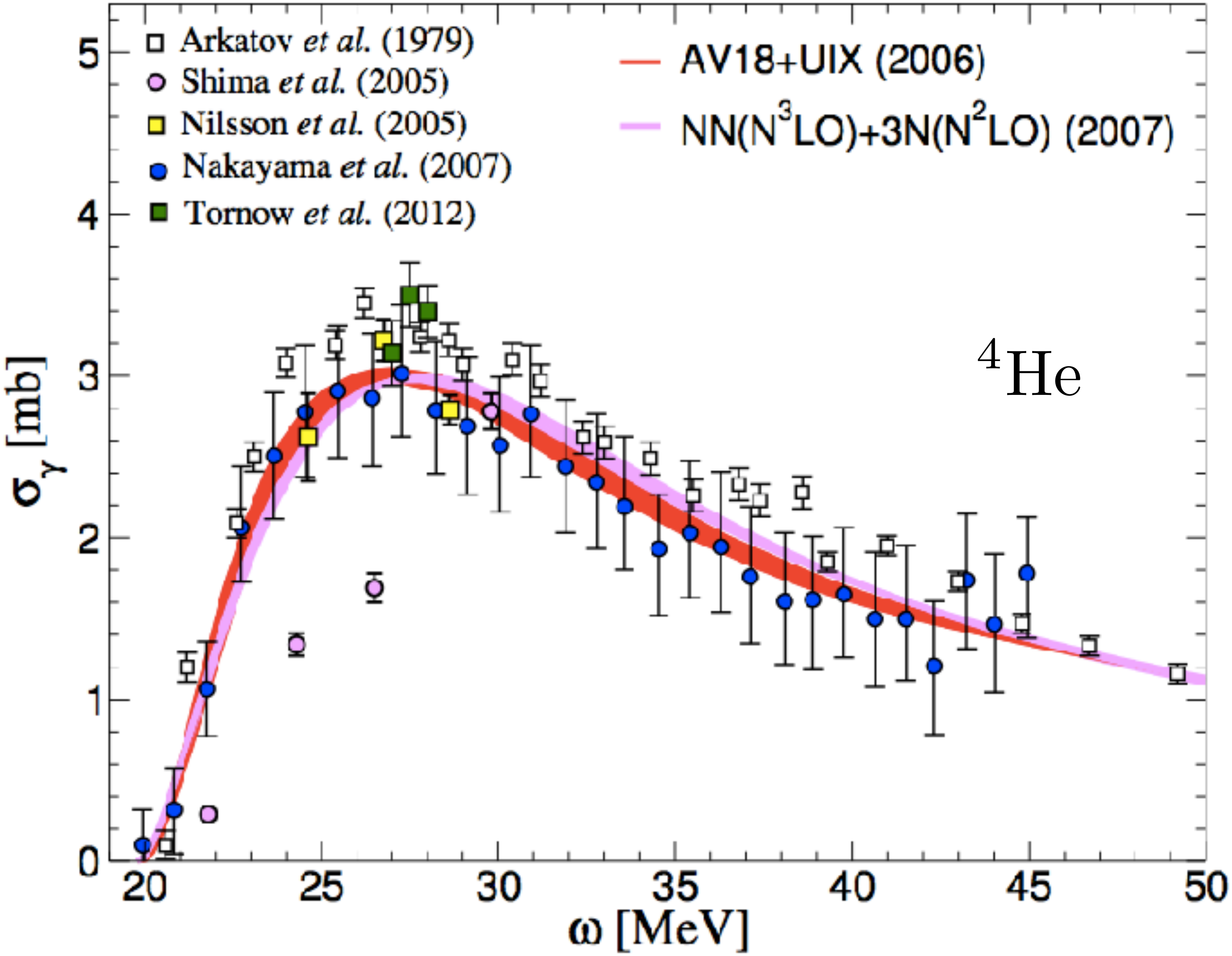


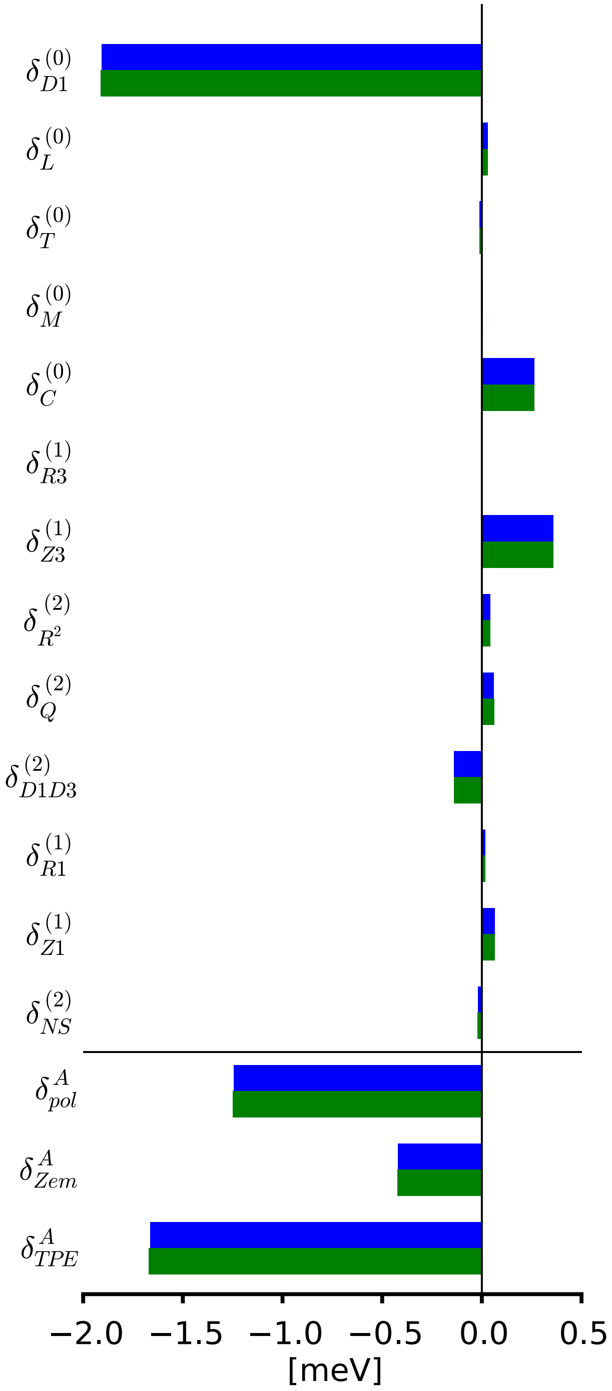
$$\delta_{D1} \rightarrow S_{\text{D1}}(\omega)$$

$$S_{\text{D1}}(\omega) = \frac{9}{16\pi^3 \alpha \omega Z^2} \sigma_\gamma(\omega)$$

S.B. and Saori Pastore, Journal of Physics G.: Nucl. Part. Phys. **41**, 123002 (2014)

Use these technology to analyze muonic atoms

Muonic Deuterium



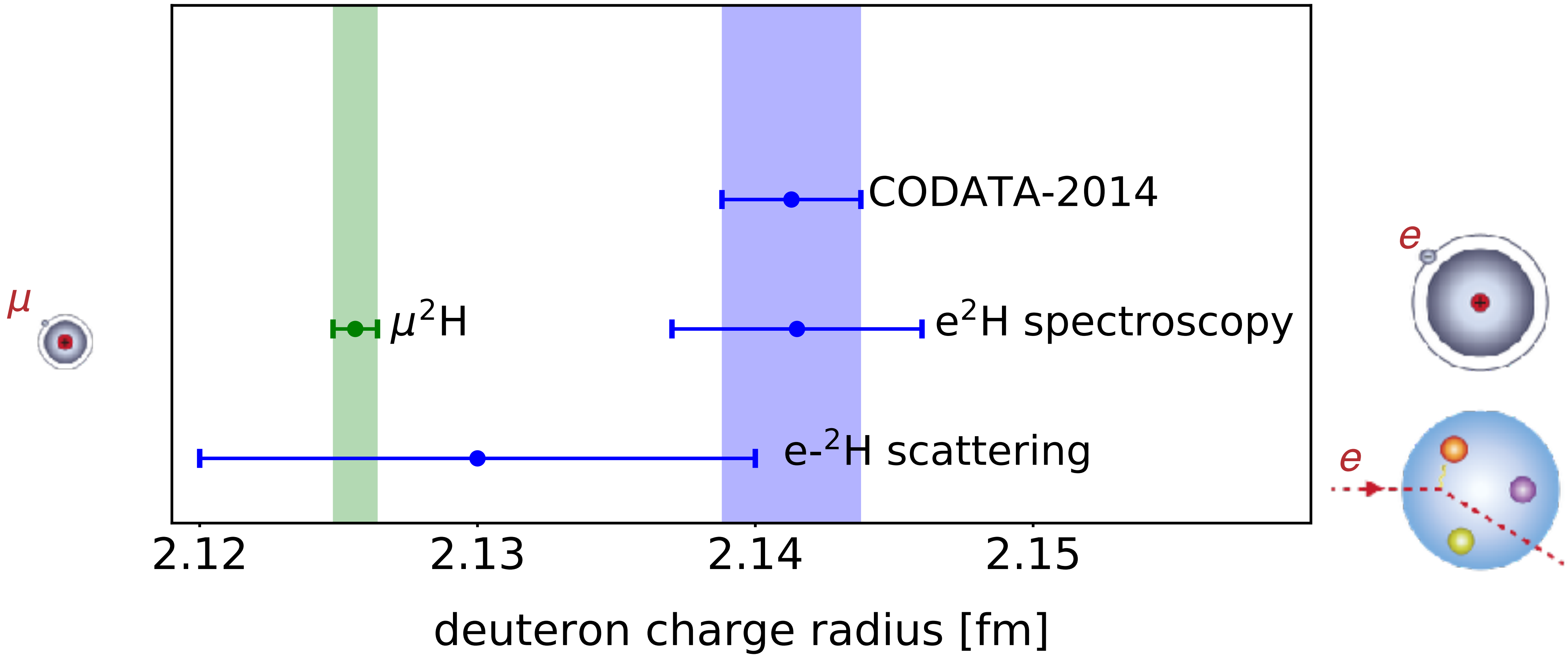
	AV18	χEFT
$\delta_{D1}^{(0)}$	-1.907	-1.912
$\delta_{L}^{(0)}$	0.029	0.029
$\delta_{T}^{(0)}$	-0.012	-0.012
$\delta_{M}^{(0)}$	0.003	0.003
$\delta_{C}^{(0)}$	0.262	0.262
$\delta_{R3}^{(1)}$	-	-
$\delta_{Z3}^{(1)}$	0.357	0.359
$\delta_{R^2}^{(2)}$	0.042	0.041
$\delta_{Q}^{(2)}$	0.061	0.061
$\delta_{D1D3}^{(2)}$	-0.139	-0.139
$\delta_{R1}^{(1)}$	0.017	0.017
$\delta_{Z1}^{(1)}$	0.064	0.064
$\delta_{NS}^{(2)}$	-0.020	-0.021
δ_{pol}^{A}	-1.243	-1.248
δ_{Zem}^{A}	-0.421	-0.423
δ_{TPE}^{A}	-1.664	-1.671

O.J. Hernandez et al, Phys. Lett. B **736**, 344 (2014)

AV18 in agreement with Pachucki (2011)+ Pachucki, Wienczek (2015)

Deuteron radius puzzle

Pohl et al, Science **353**, 669 (2016)



$$\Delta E_{2S-2P} = \delta_{\text{QED}} + \mathcal{A}_{\text{OPE}} \langle r_c^2 \rangle + \delta_{\text{TPE}}$$

Hernandez et al., PLB **736**, 334 (2014)

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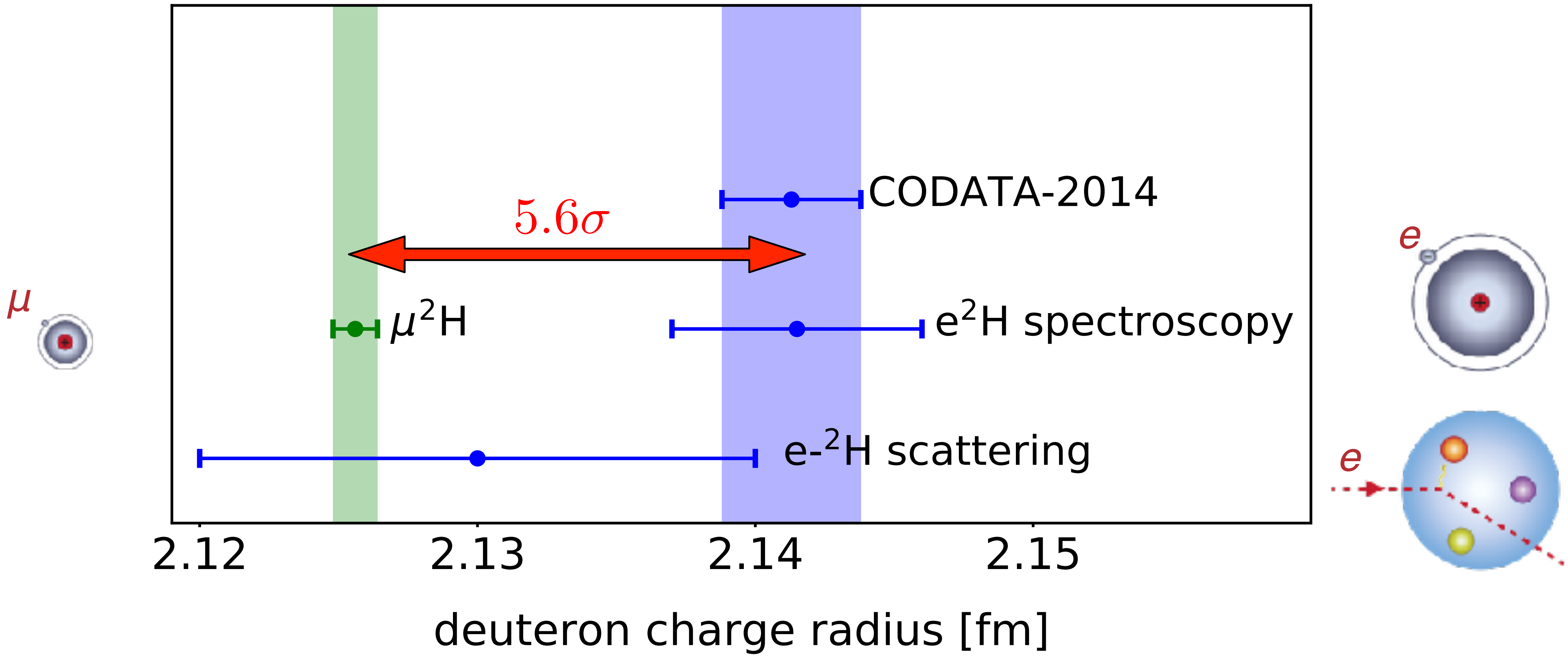

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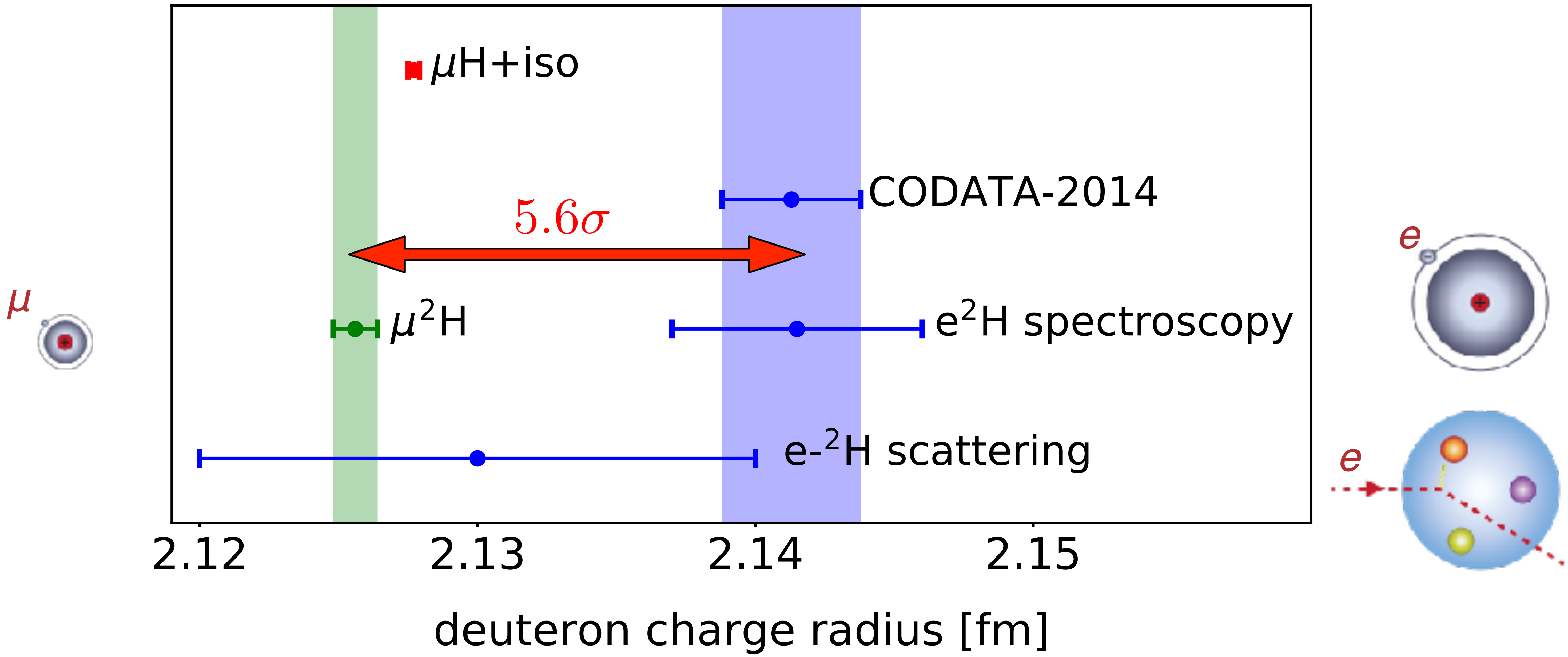

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μH+iso: r_p from μH and deuterium isotopic shift r^2_d -r^2_p: Parthey et al., PRL **104** 233001 (2010)

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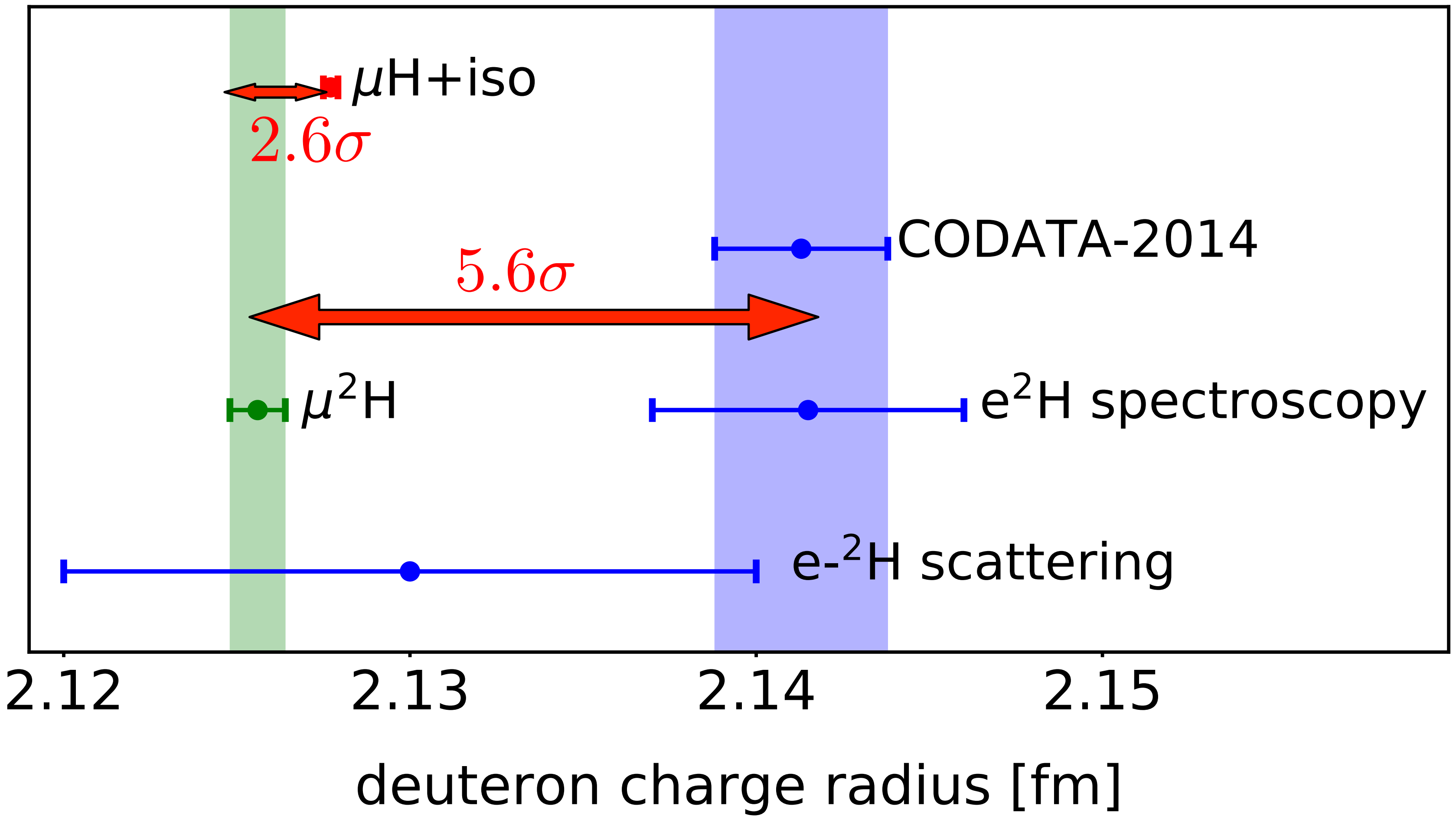
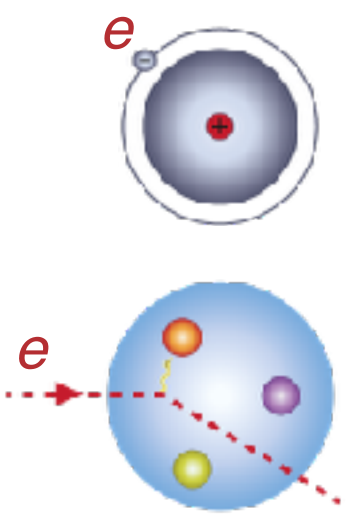
Pohl et al, Science **353**, 669 (2016)





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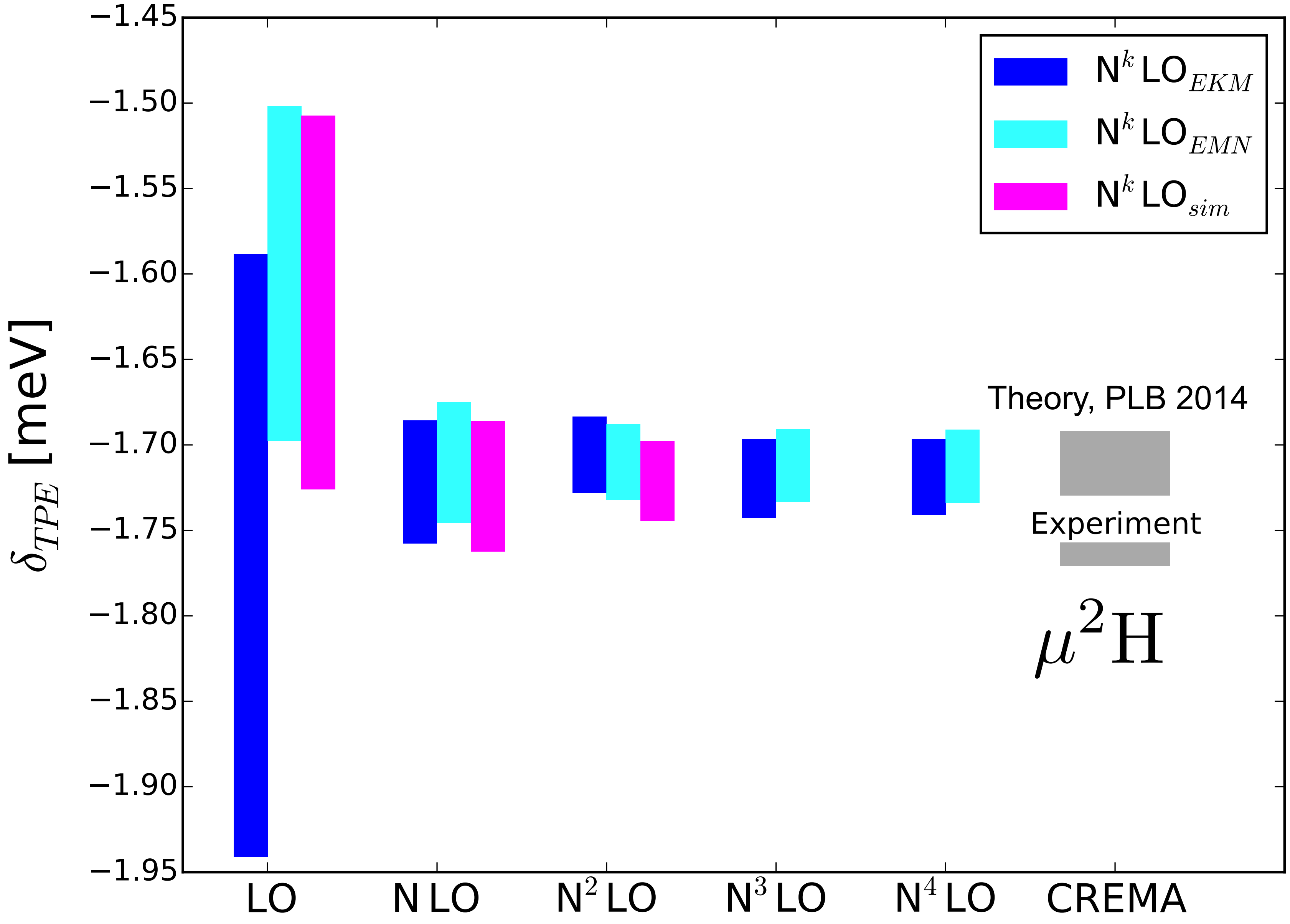
Order-by-order chiral expansion

Statistical and systematic uncertainty analysis

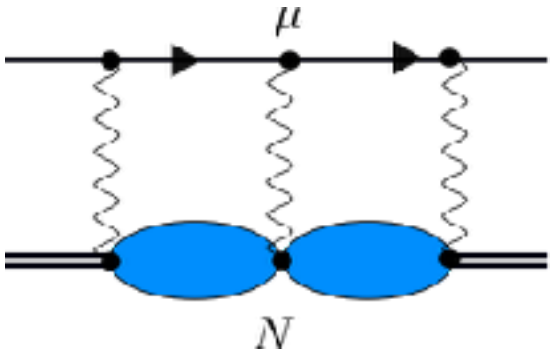
O.J. Hernandez et al, Phys. Lett. B **778**, 377 (2018)



Only sightly mitigate the “small” proton radius puzzle (2.6 to 2 σ)

Higher order corrections in α

Three-photon exchange



Pachucki et al., Phys. Rev. A **97** 062511 (2018)

$(Z\alpha)^6$ correction, negligible

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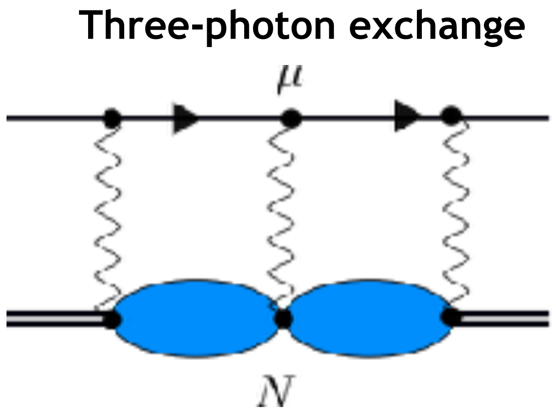

Pachucki et al., Phys. Rev. A **97** 062511 (2018)

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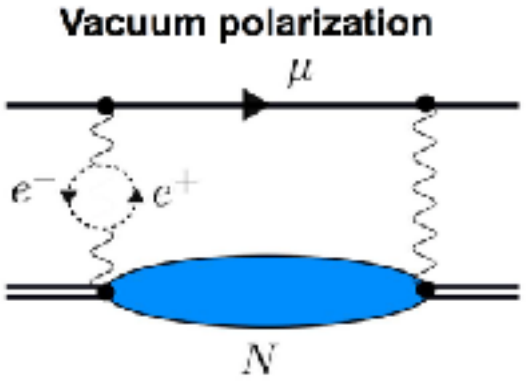


One the many α^6 corrections, supposedly the largest

Kalinowski, Phys. Rev. A **99** 030501 (2019)

$$\delta_{\rm TPE} = -1.750^{+14}_{-16}\ {\rm meV}$$ Theory

$$\delta_{\rm TPE} = -1.7638(68)\ {\rm meV}$$ Exp

Consistent within 1σ
solves the small deuteron-radius puzzle

Large deuteron-radius puzzle still unsolved!
New data on electron scattering expected from MAMI and from the future MESA

Uncertainties quantifications

Uncertainties sources

- Numerical
- Nuclear model
- Nucleon-size
- Truncation of multiples
- η-expansion
- expansion in $Z\alpha$

Impact of ab initio theory

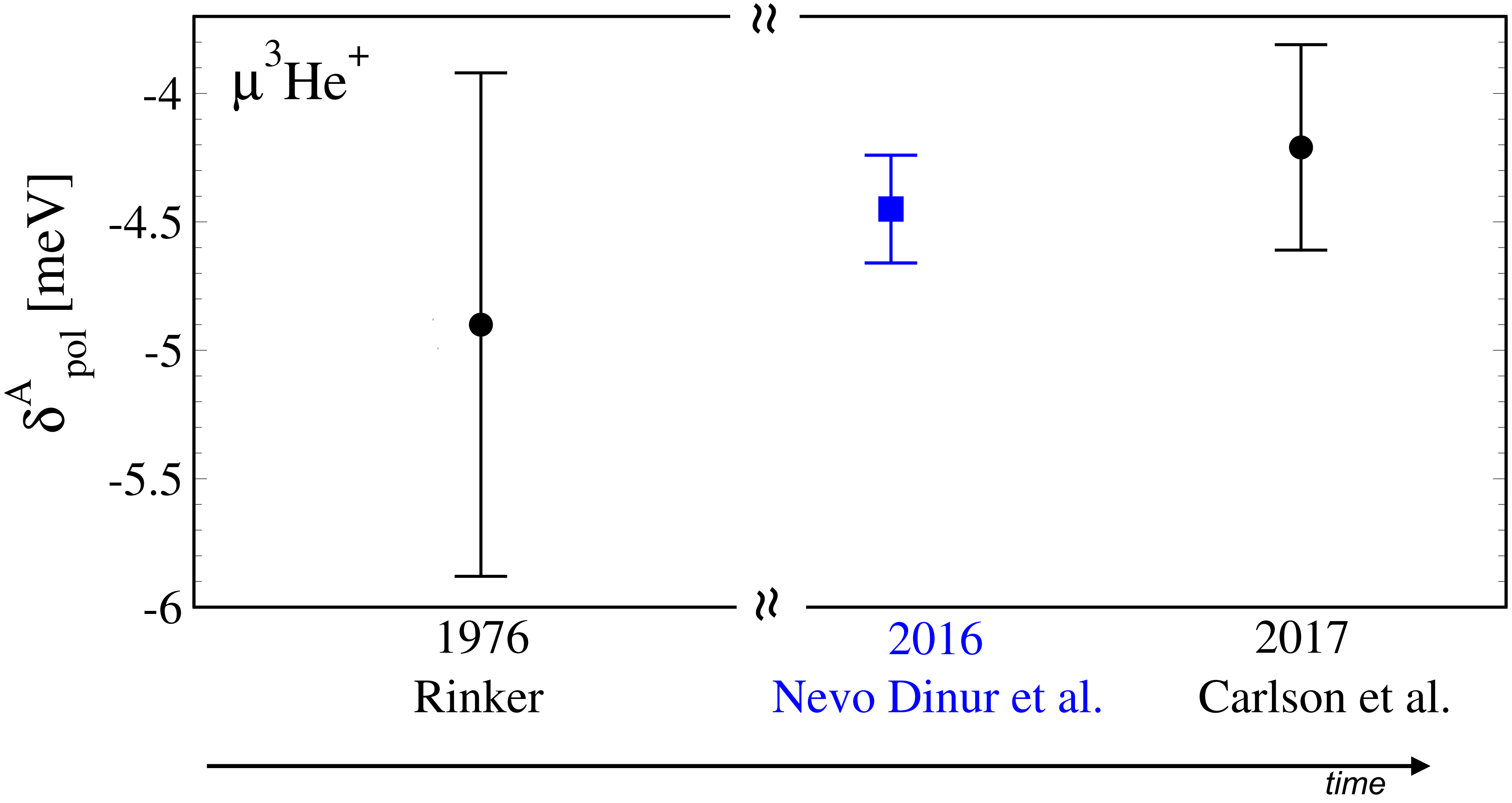
- Reduction of Uncertainties -

Atom	Exp uncertainty on ΔE_{2S-2P}	Uncertainty on TPE prior to the discovery of the puzzle	Uncertainty on TPE: ab initio
μ^2H	0.003 meV	0.03* meV	0.02 meV
μ^3He$^+$	0.08 meV	1 meV	0.3 meV
μ^4He$^+$	0.06 meV	0.6 meV	0.4 meV
$\mu\ ^{6,7}$Li^{++}	0.7 meV	4 meV	

*Leidemann, Rosenfelder ’95 using few-body methods

Impact of ab initio theory

- Reduction of Uncertainties -



C.Ji et al., JPG: Part. Nucl. **45,** 093002 (2018)

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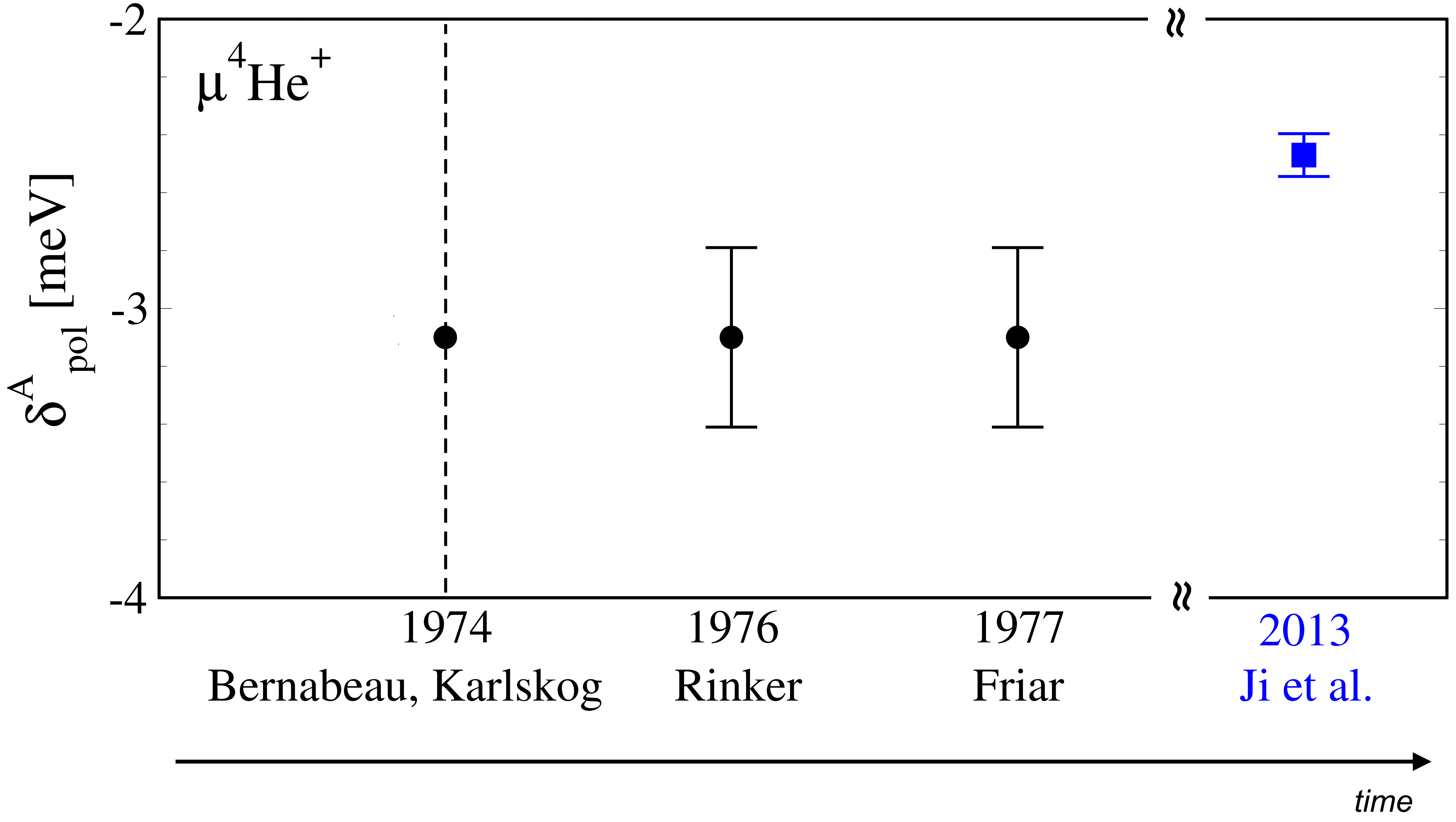

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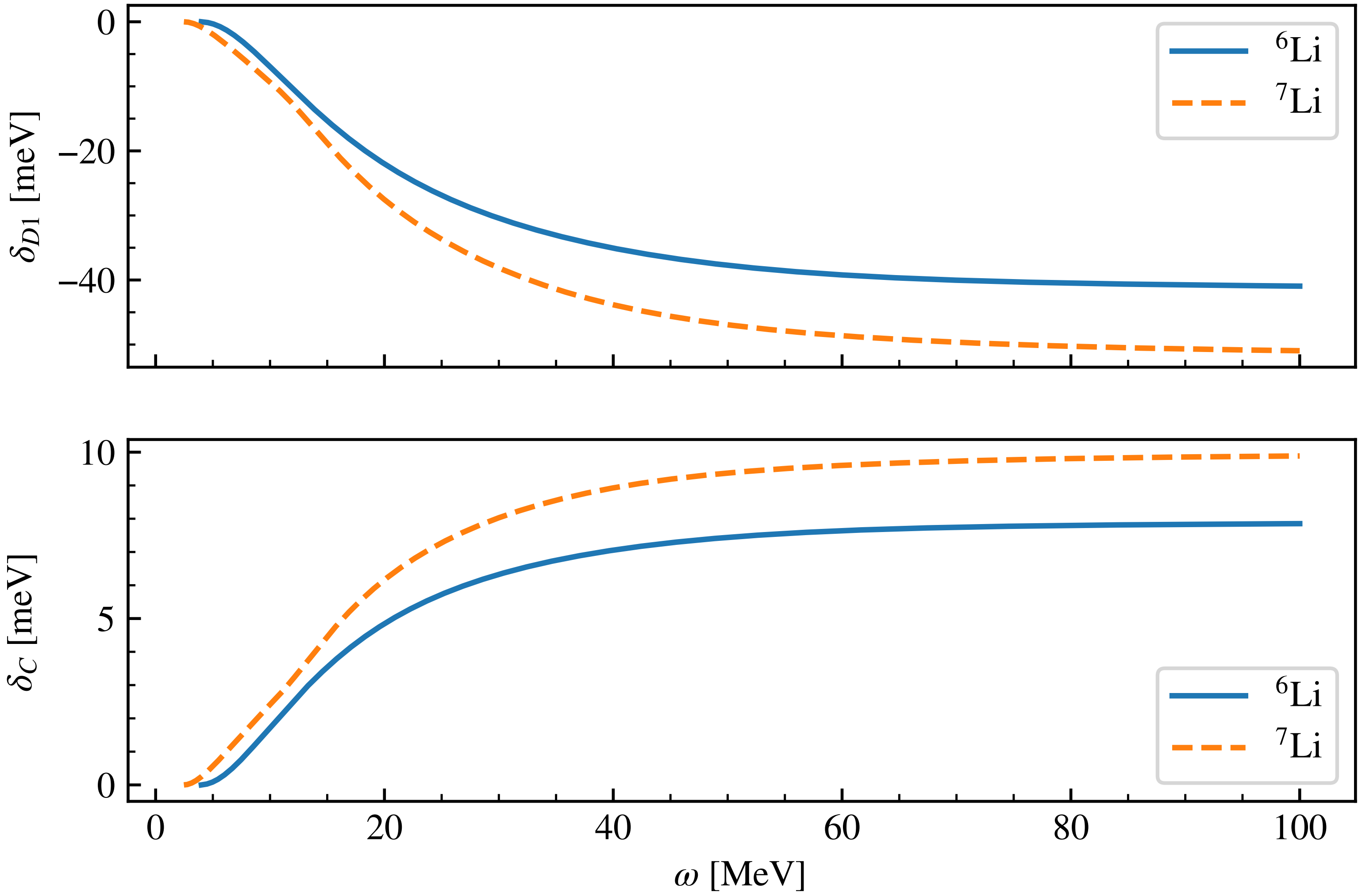
Muonic Lithium

Simone Li Muli
PhD candidate
in Mz

$$\delta^{(0)}_{\mathrm{D1}} \propto \int_0^\infty d\omega \sqrt{\frac{2m_r}{\omega}} S_{\mathrm{D1}}(\omega)$$

$$\delta^{(0)}_{\mathrm{C}} \propto \int_0^\infty d\omega \frac{m_r}{\omega} \ln \frac{2(Z\alpha)^2 m_r}{\omega} S_{\mathrm{D1}}(\omega)$$



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A. Poggialini,
S.B,

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With AV4’

Semi realistic

potential

Muonic Lithium

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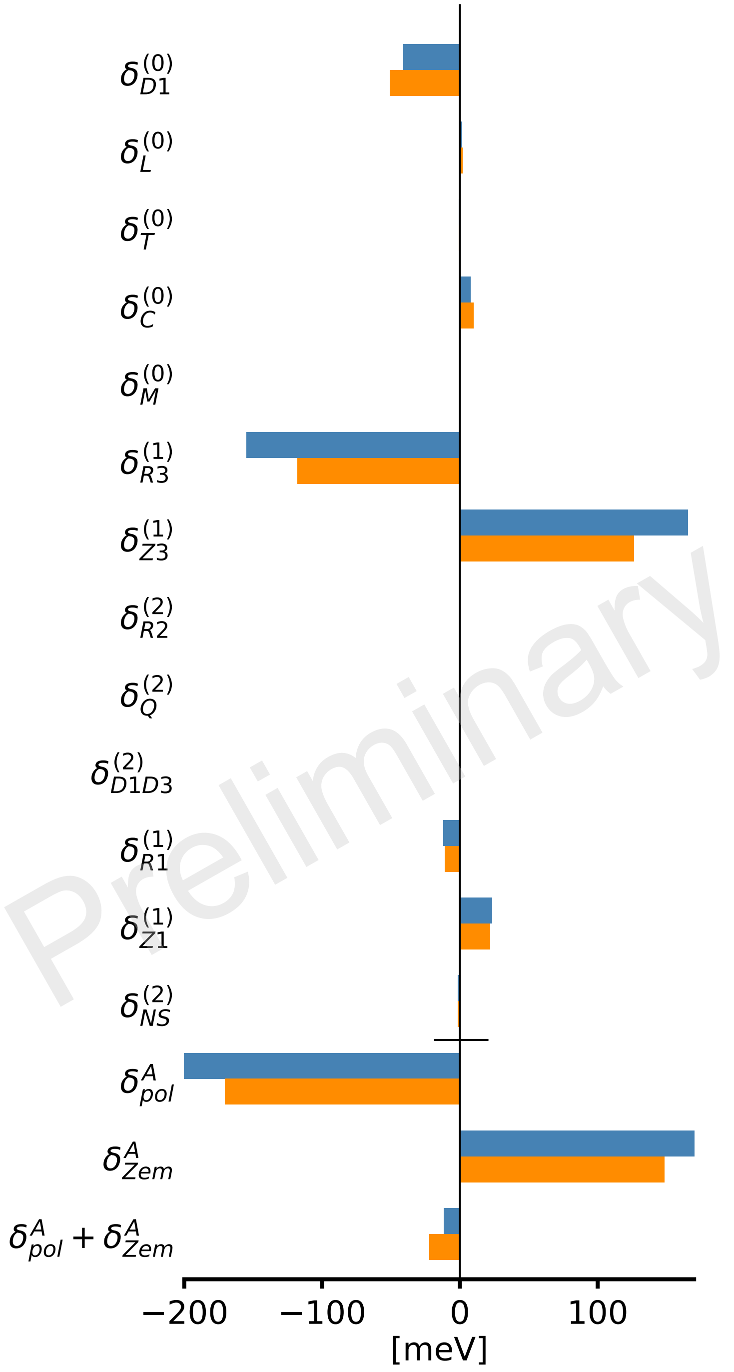

	μ-^{6}Li^{2+}	μ-^{7}Li^{2+}
$\delta_{D1}^{(0)}$	-41.0	-51.0
$\delta_{L}^{(0)}$	1.66	2.04
$\delta_{T}^{(0)}$	-0.75	-0.92
$\delta_{C}^{(0)}$	7.85	9.89
$\delta_{M}^{(0)}$	-	-
$\delta_{R3}^{(1)}$	-155.0	-118.0
$\delta_{Z3}^{(1)}$	165.4	126.5
$\delta_{R2}^{(2)}$	-	-
$\delta_{Q}^{(2)}$	-	-
$\delta_{D1D3}^{(2)}$	-	-
$\delta_{R1}^{(1)}$	-12.0	-11.0
$\delta_{Z1}^{(1)}$	23.47	22.03
$\delta_{NS}^{(2)}$	-1.41	-1.75
δ_{pol}^{A}	-200.65	-170.74
δ_{Zem}^{A}	188.87	148.53
$\delta_{pol}^{A}+\delta_{Zem}^{A}$	-11.78	-22.21

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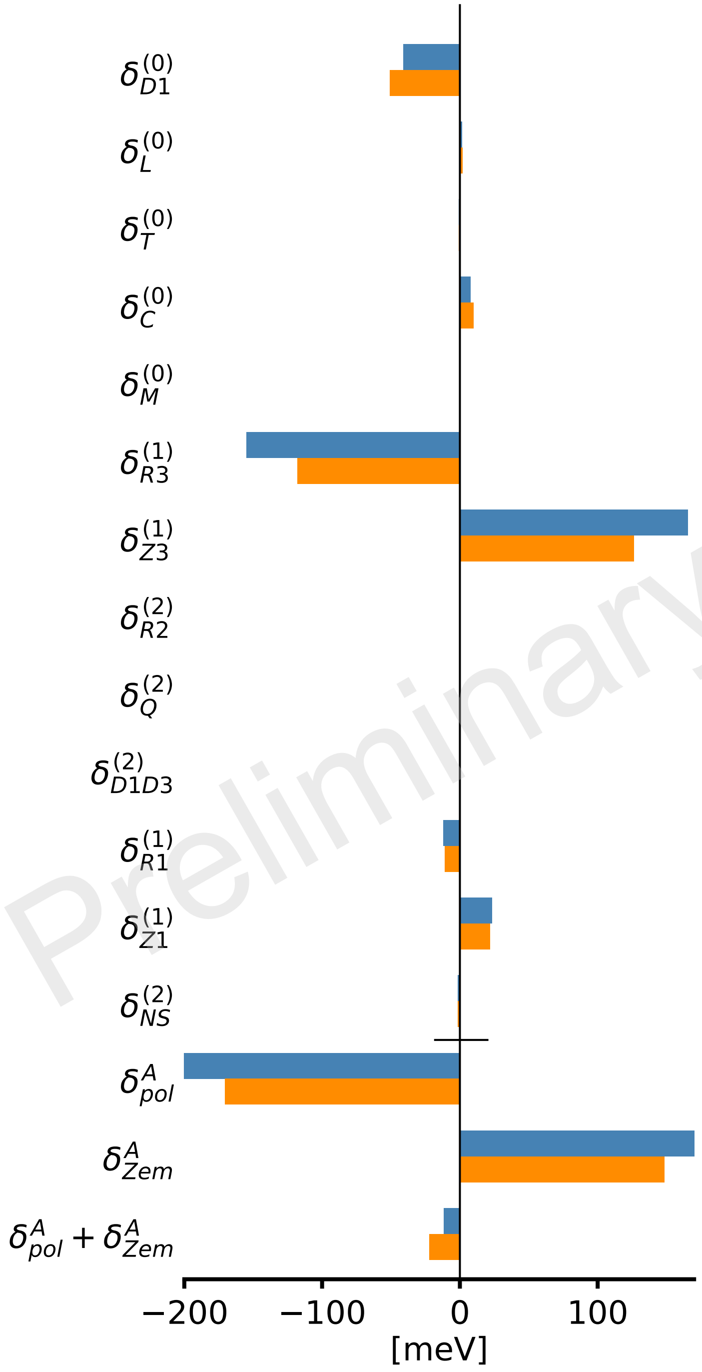

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-15 ± 4 -21 ± 4 Drake et al., PRA (1985)

Summary and Outlook

- Ab initio calculations have allowed to substantially reduce uncertainties
- Independently on the nature of the puzzle, these calculations are needed to support any spectroscopic measurement with muonic atoms
- In the future we will investigate the Lamb shift in muonic lithium atoms and address the hyperfine splitting

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Thanks to my collaborators

N.Barnea, **O.J. Hernandez**, **C.Ji**, **S. Li Muli**, **N. Nevo Dinur**, **A. Poggialini**

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Thank you for your attention!